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Optimizing Global Injectivity for Constrained Parameterization

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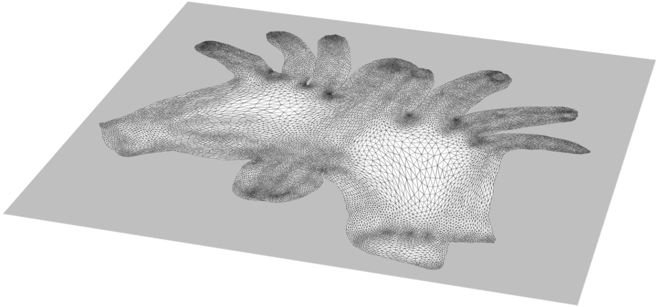
Mapping Meshes to Plane

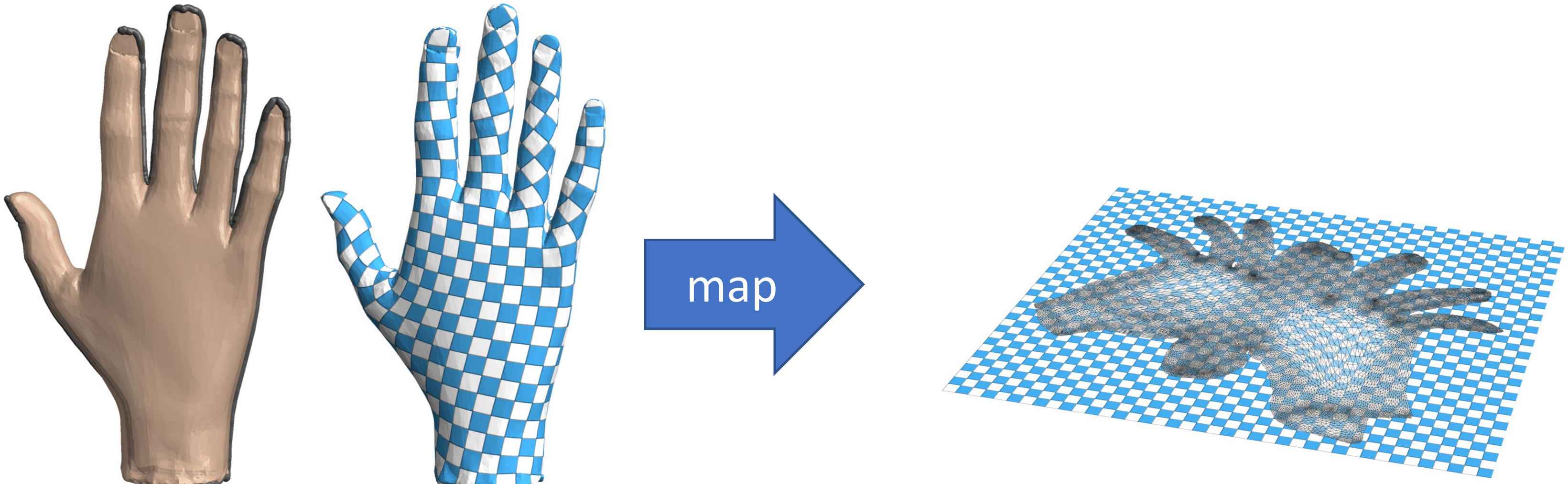
triangle mesh

map



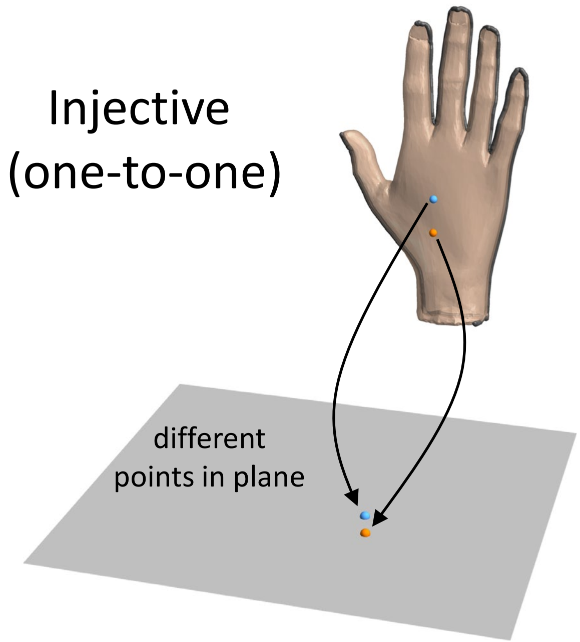
2D plane

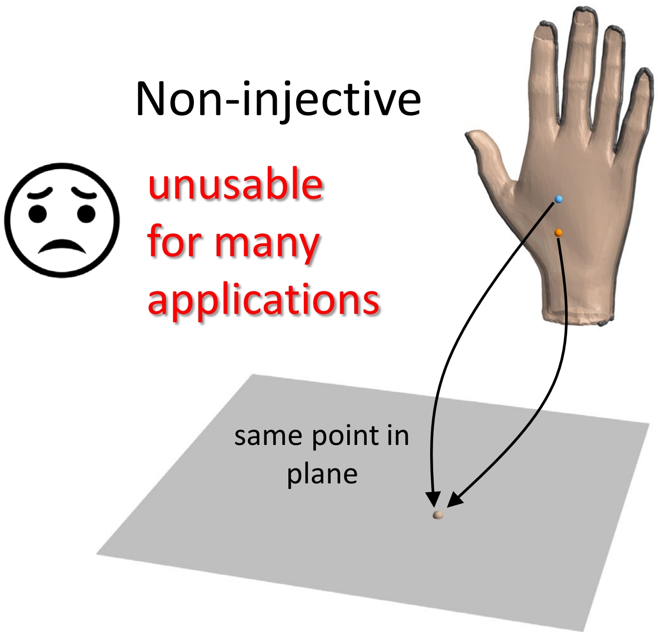
Application: Texture Mapping



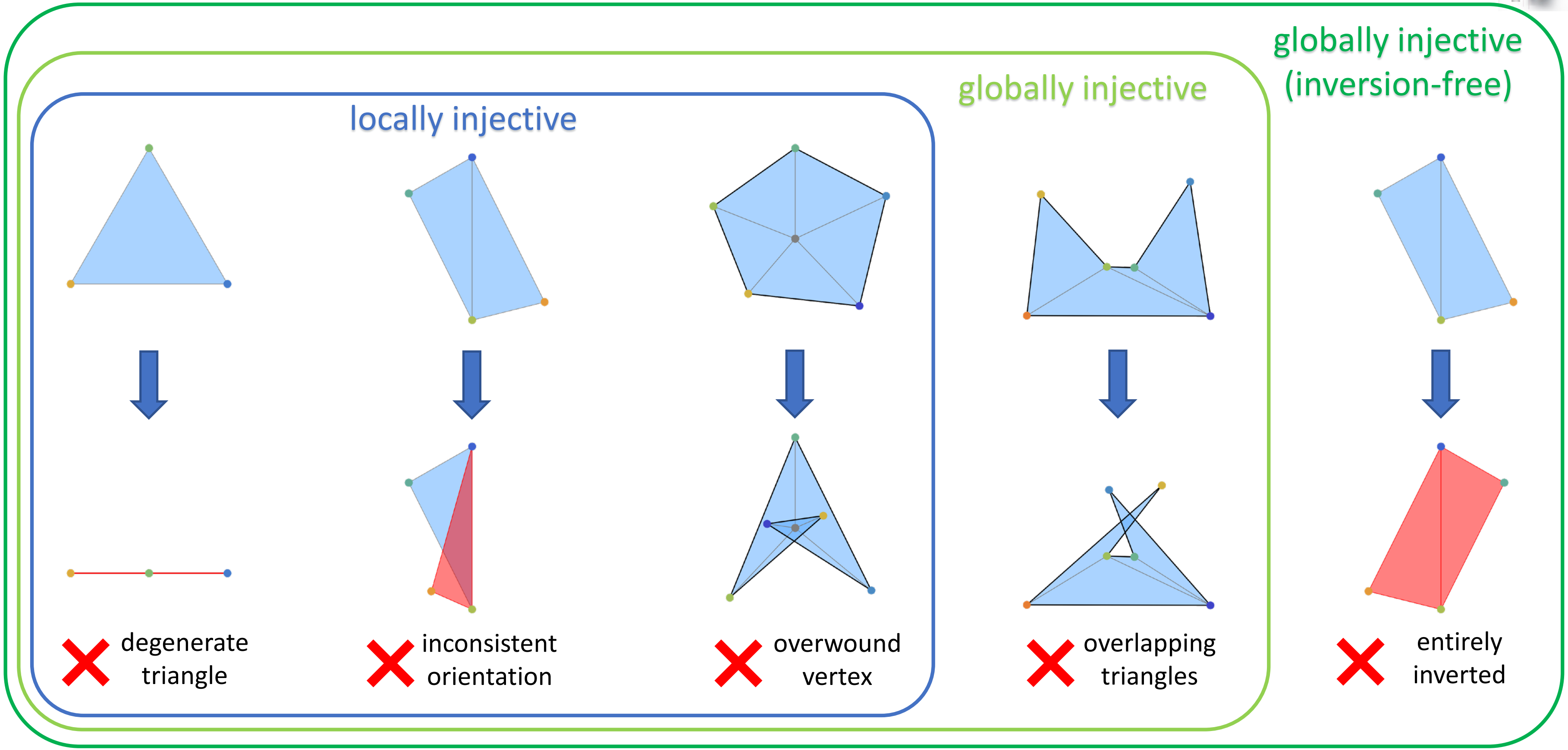
2D Texture

Injective (one-to-one) Mapping

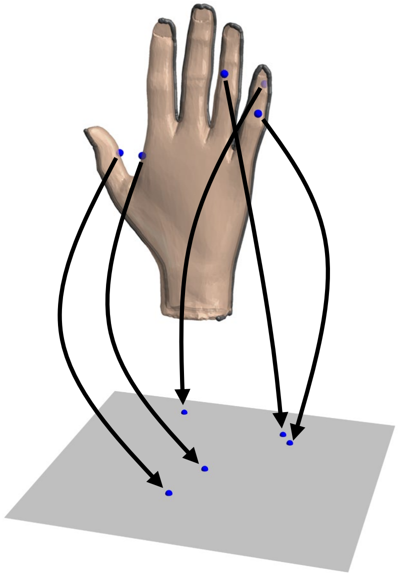




Injectivity Criteria



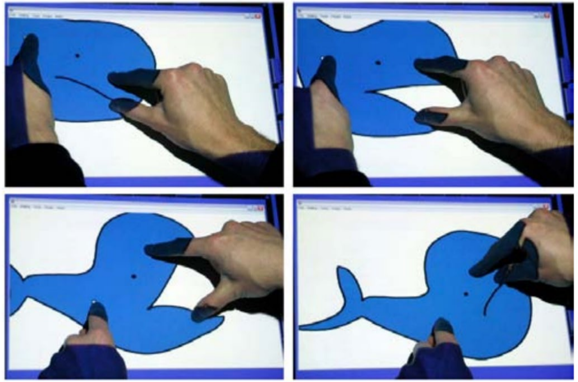
Mapping under Positional Constraints



positional constraints

constrained texture mapping

[Lévy 2001]



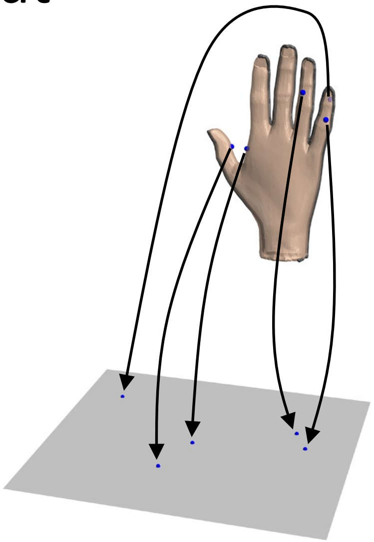
handle-based deformation

[Igarashi et al. 2005]

Goal: Injective Mapping under Positional Constraints

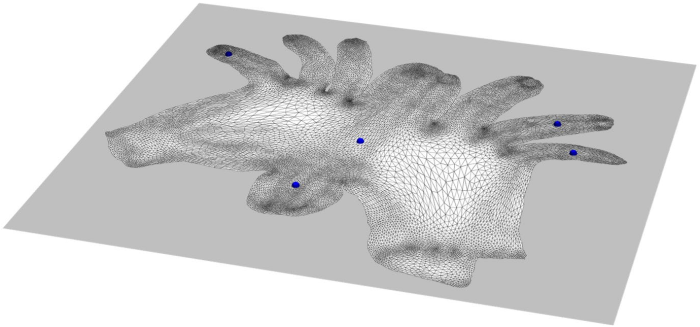
Input

(1) source mesh



(2) positional constraints

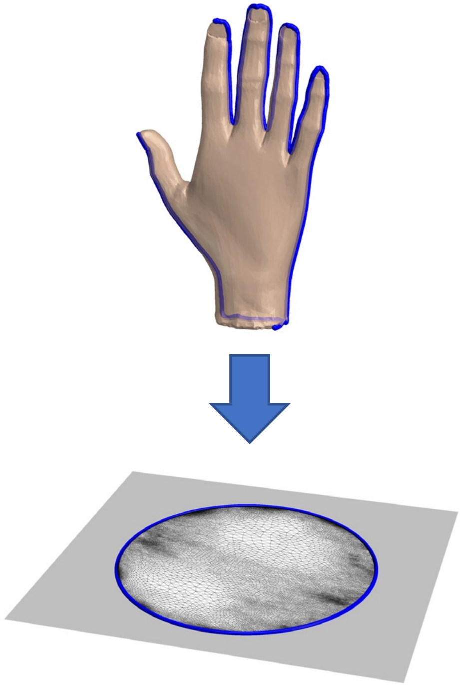
Output



injective mapping satisfying positional constraints

Injective Mapping

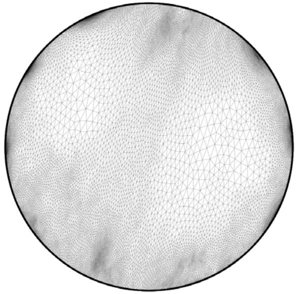
- Tutte embedding [Tutte 1963]
    - Convex boundary and no positional constraints



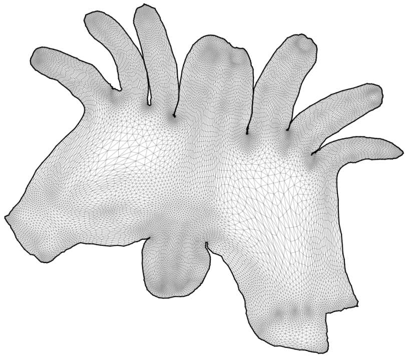
Tutte embedding
[Tutte 1963]

Injective Mapping

- Tutte embedding [Tutte 1963]
    - Convex boundary and no positional constraints
- Maintenance methods
    - [Hormann and Greiner 2000], [Schüller et al. 2013], [Smith and Schaefer 2015], AMIPS [Fu et al. 2015], [Liu et al. 2016], SLIM [Rabinovich et al. 2017], CM [Shtengel et al. 2017], [Claici et al. 2017], SCAF[Jiang et al. 2017], BCQN [Zhu et al. 2018], [Liu et al. 2018], [Su et al. 2020], IDP [Fang et al. 2021]
    - Require injective maps to initialize



injective initial map



SCAF
[Jiang et al. 2017]

Injective Mapping

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  - Convex boundary and no positional constraints
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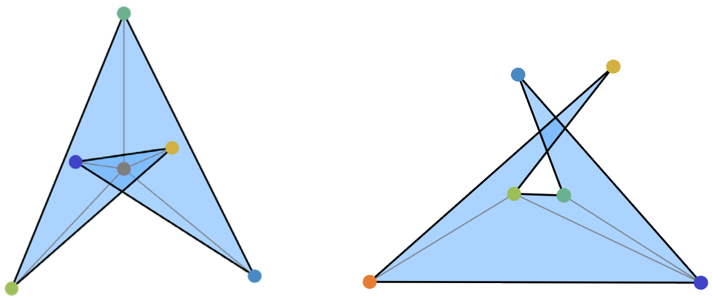
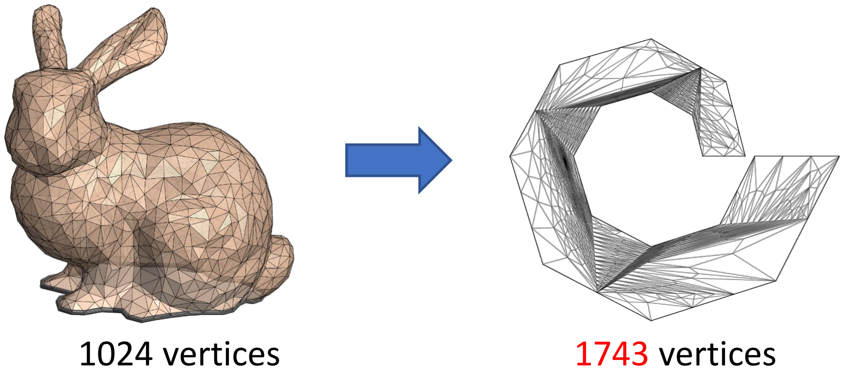
  - Require injective maps to initialize
- Inversion-free methods
  - [Aigerman and Lipman 2013], LBD [Kovalsky et al. 2015], SA [Fu and Liu 2016], FF [Su et al. 2019], [Hefetz et al. 2019], TLC [Du et al. 2020]
  - Cannot avoid overlapping triangles



overlapping

Injective Mapping

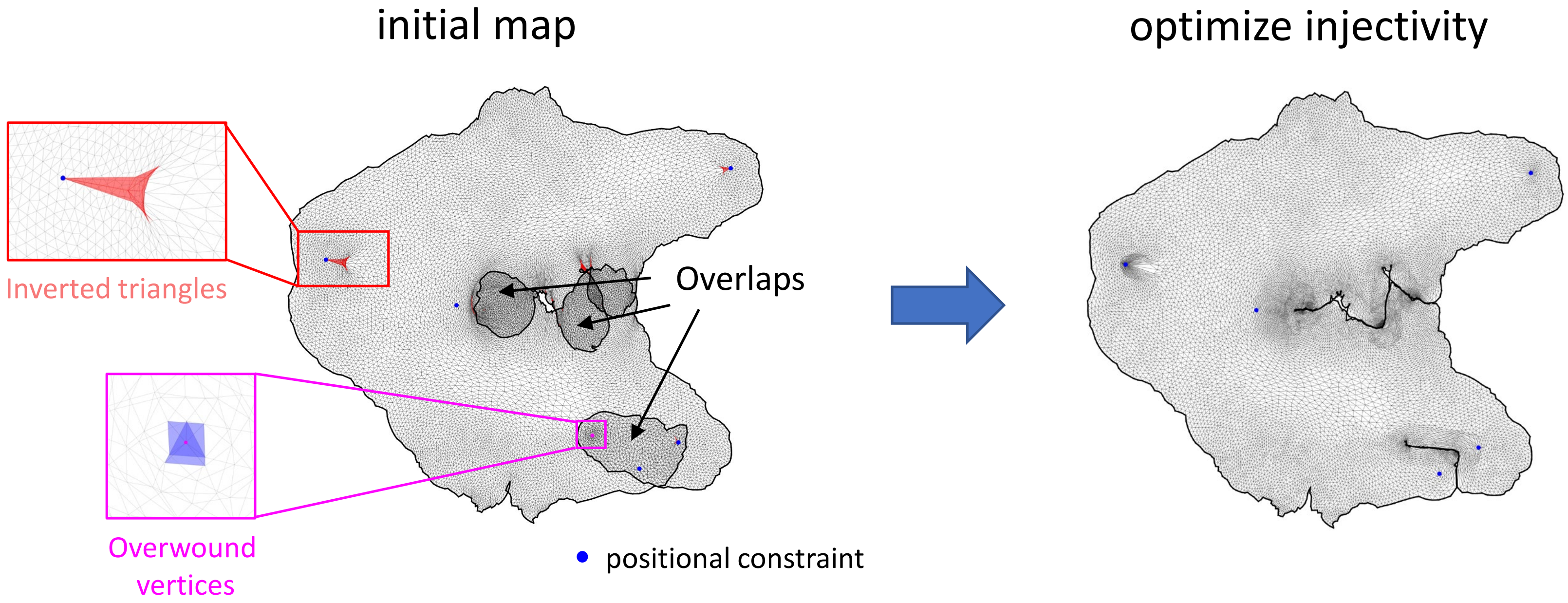
- Tutte embedding [Tutte 1963]
  - Convex boundary and no positional constraints
- Maintenance methods
  - [Hormann and Greiner 2000], [Schüller et al. 2013], [Smith and Schaefer 2015], AMIPS [Fu et al. 2015], [Liu et al. 2016], SLIM [Rabinovich et al. 2017], CM [Shtengel et al. 2017], [Claici et al. 2017], SCAF[Jiang et al. 2017], BCQN [Zhu et al. 2018], [Liu et al. 2018], [Su et al. 2020], IDP [Fang et al. 2021]
  - Require injective maps to initialize
- Inversion-free methods
  - [Aigerman and Lipman 2013], LBD [Kovalsky et al. 2015], SA [Fu and Liu 2016], FF [Su et al. 2019], [Hefetz et al. 2019], TLC [Du et al. 2020]
  - Cannot avoid overlapping triangles
- Remeshing methods
  - [Eckstein et al. 2001], Matchmaker [Kraevoy et al. 2003], [Lee et al. 2008], [Agarwal et al. 2008], [Weber and Zorin 2014], [Gu et al. 2018], [Shen et al. 2019]
  - May change the mesh structure



Progressive Embedding
[Shen et al. 2019]

Contribution

- First method for computing globally injective maps under positional constraints (without changing the mesh structure)

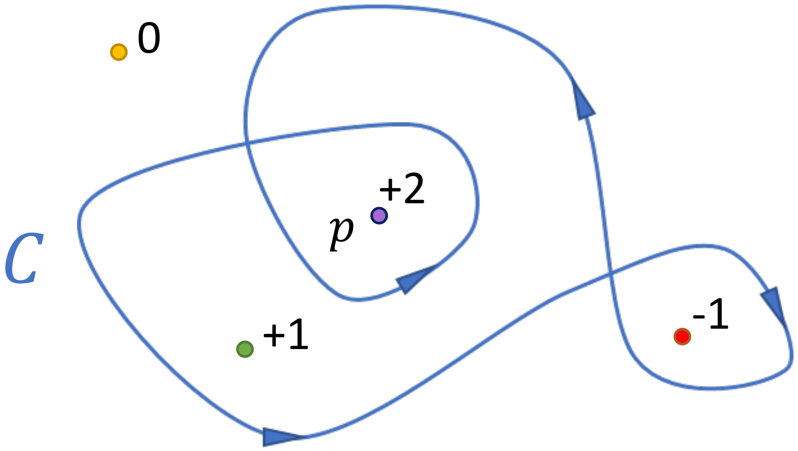


Contribution

- New energy for promoting injectivity
  - Captures both inverted and overlapping areas
  - Smooth almost everywhere
  - Theoretical guarantees of injectivity at global minima

Background: Winding Number

- Number of times a curve C travels CCW around a point p



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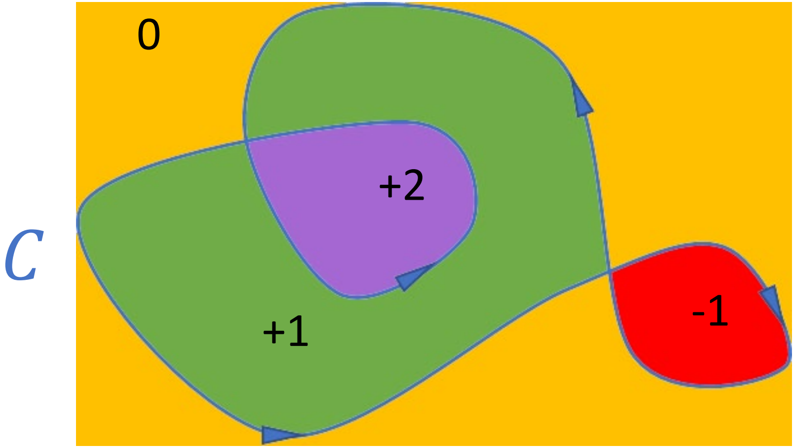
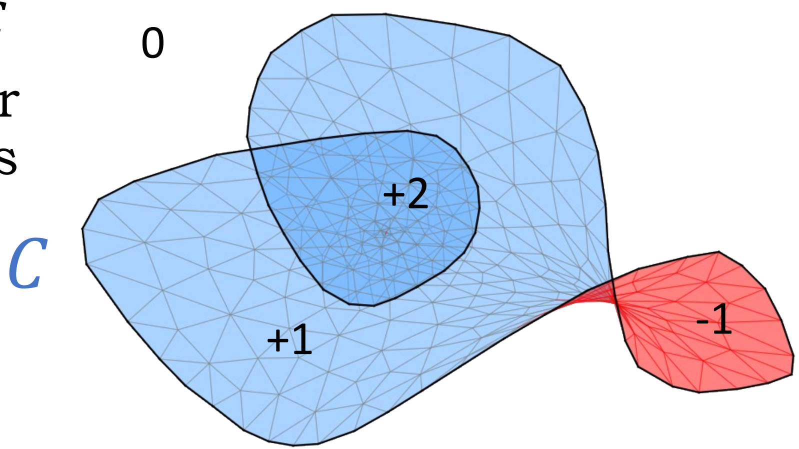


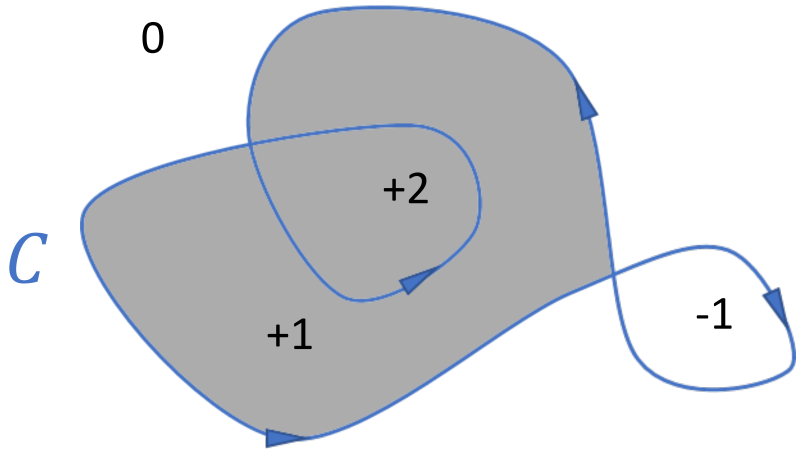
Background: Winding Number

- Consider a mesh bounded by C
- Winding number is the number of non-inverted triangles minus number of inverted triangles covering a point



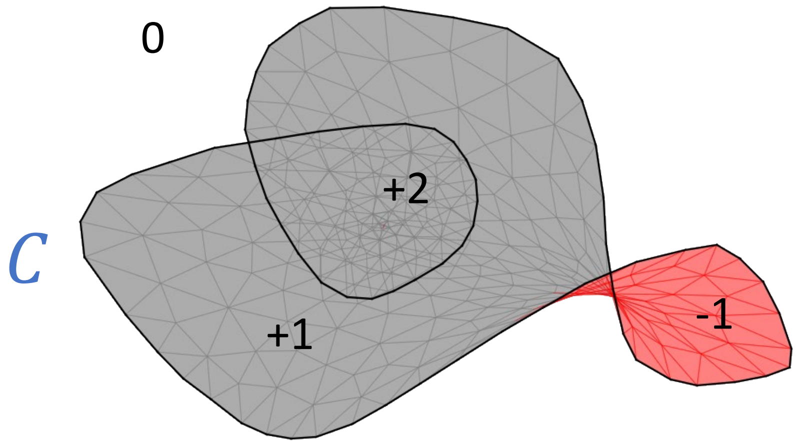
Occupancy

- The total area with positive winding numbers



Occupancy

- The total area with positive winding numbers
- The area covered by at least one non-inverted triangle for any mesh bounded by C



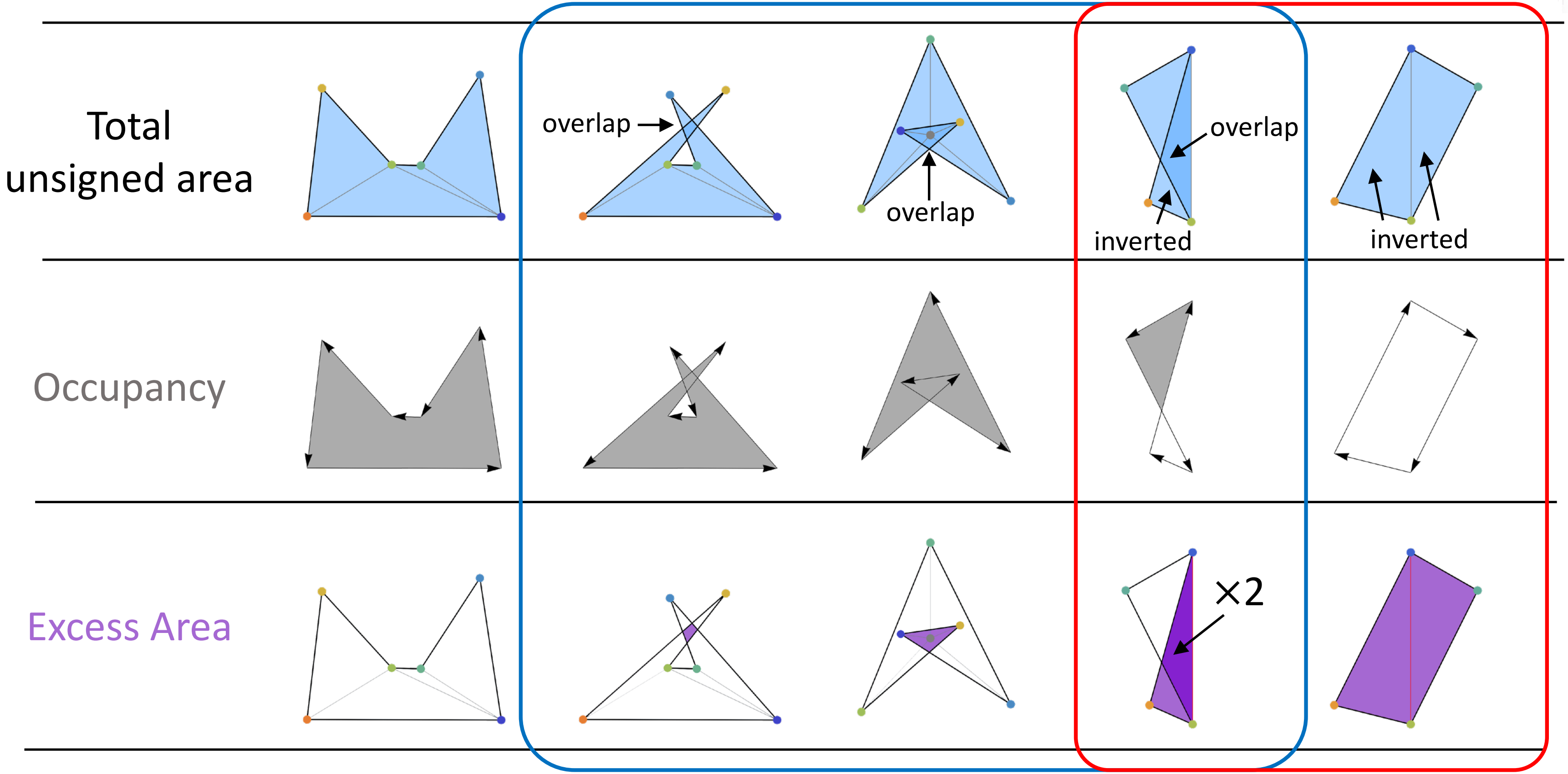
Excess Area

- Mesh T with boundary ∂T

$$\overset{\text{excess area}}{\boxed{A_{excess}(T)}} = \overset{\text{total unsigned area}}{\boxed{A_{unsigned}(T)}} - \overset{\text{occupancy}}{\boxed{O(\partial T)}}$$

- Properties
  - $\max\left(A_{overlap}, A_{inverted}\right) \leq A_{excess} \leq A_{overlap} + A_{inverted}$
  - Zero if and only if T has no overlapping or inverted triangles

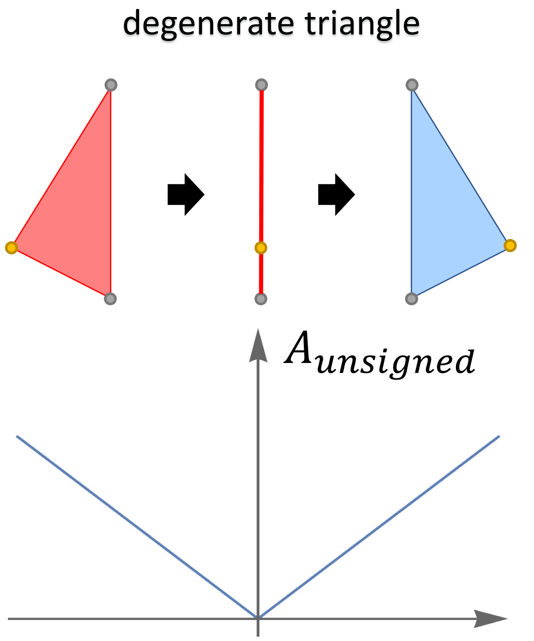
Excess Area

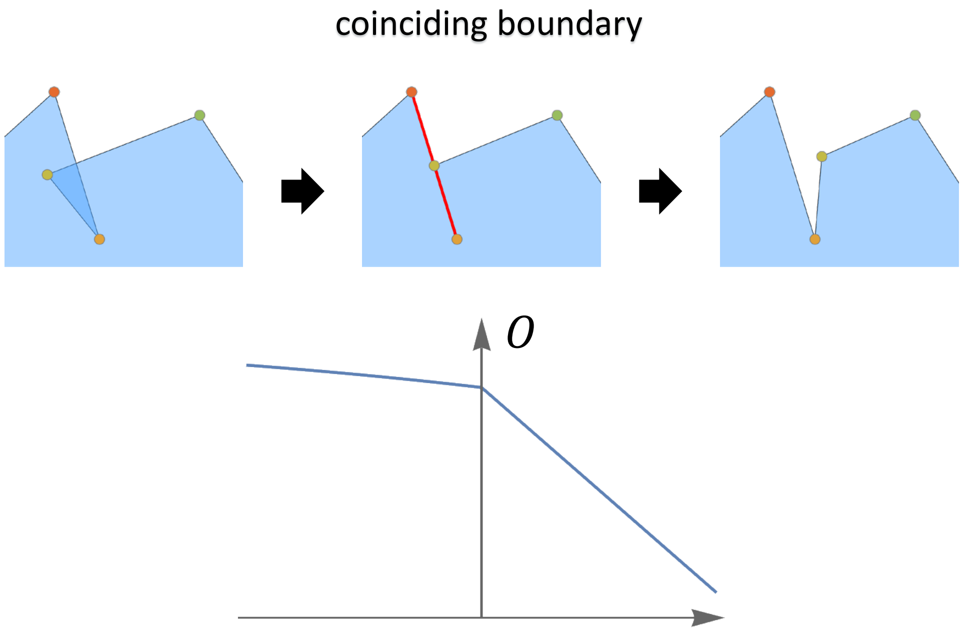

Total
unsigned area
Occupancy
Excess Area
overlap
overlap
overlap
inverted
inverted
×2

Non-smoothness

excess area | total unsigned area | occupancy

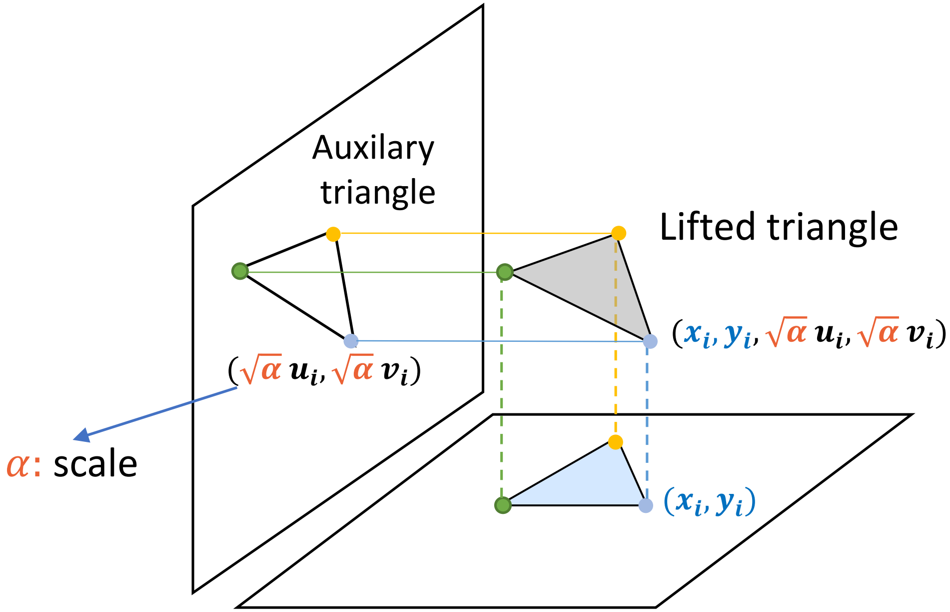
$$\boxed{A_{excess}(T)} = \boxed{A_{unsigned}(T)} - \boxed{O(\partial T)}$$

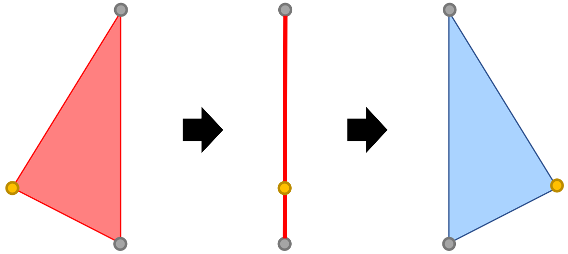


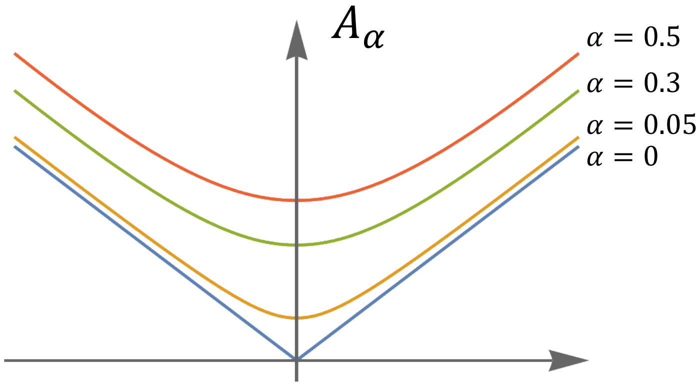


Smoothing Unsigned Area

- Total lifted Content (TLC) [Du et al. 2020]
  - Area of lifted triangles

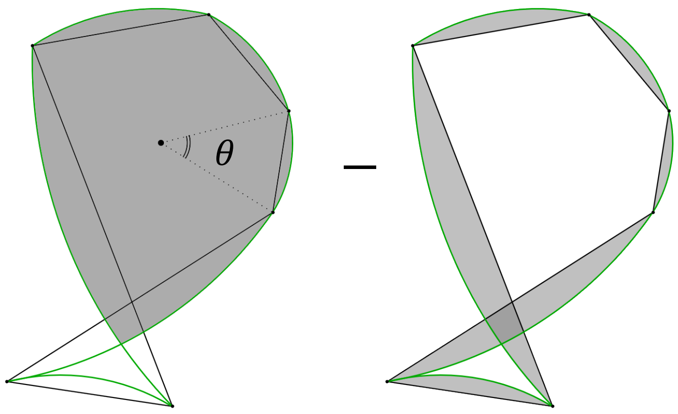


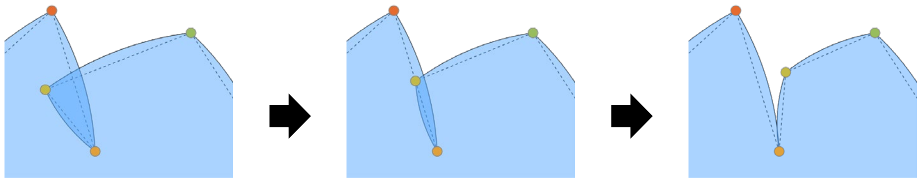


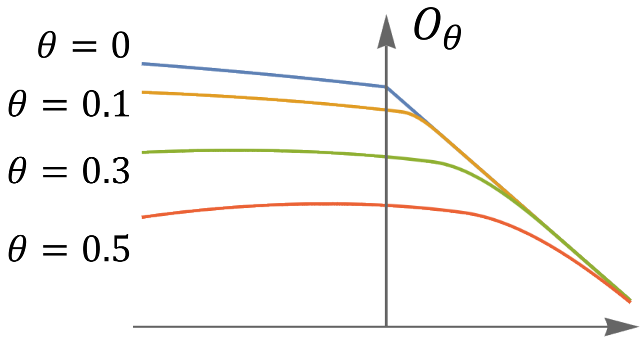


Smoothing Occupancy

- Arc Occupancy







Smooth Excess Area (SEA)

- Mesh T with boundary ∂T

$$\underbrace{\boxed{E_{\alpha,\theta}(T)}}_{\text{smooth excess area}} = \underbrace{\boxed{A_\alpha(T)}}_{\text{total lifted content}} - \underbrace{\boxed{O_\theta(\partial T)}}_{\text{arc occupancy}}$$

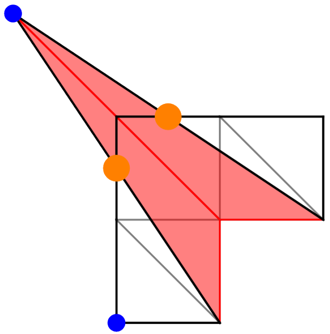
- Properties
  - For any $\alpha > 0, \theta > 0$, SEA $\geq \max(A_{overlap}, A_{inverted})$
  - For sufficiently small α and θ, the minima of SEA is *locally injective with arbitrarily small overlapping area*

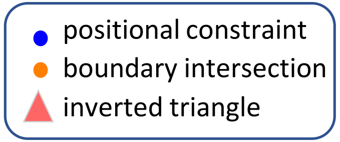
Optimizing SEA

- Quasi-Newton (L-BFGS)
- Termination Criteria
    - Map is globally injective
        - No inverted/degenerate triangles and no boundary intersection [Lipman 2014]
    - Reaches a max #iterations (e.g., 10 000)

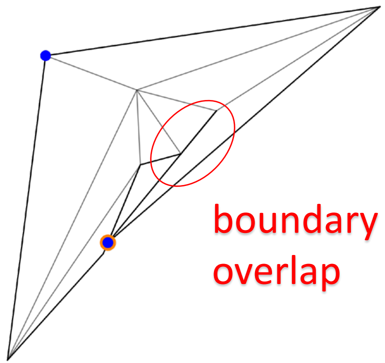
Handcrafted Examples

initial map

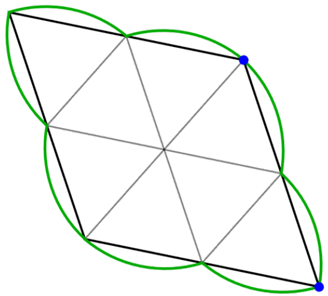




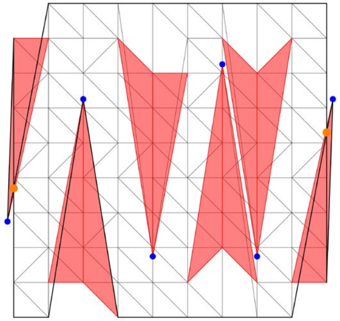
TLC – occupancy
$(\alpha = 10^{-4})$



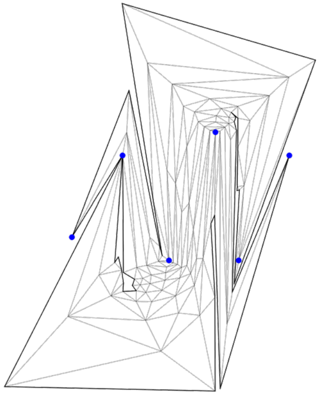
TLC — arc occupancy
$(\alpha = 10^{-4}, \theta = 1.0)$



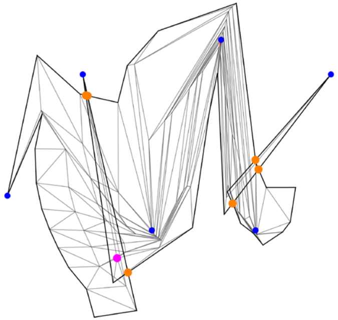
Handcrafted Examples



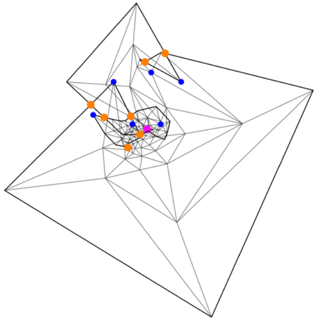
initial
map



ours
$(\alpha = 10^{-4}, \theta = 0.1)$



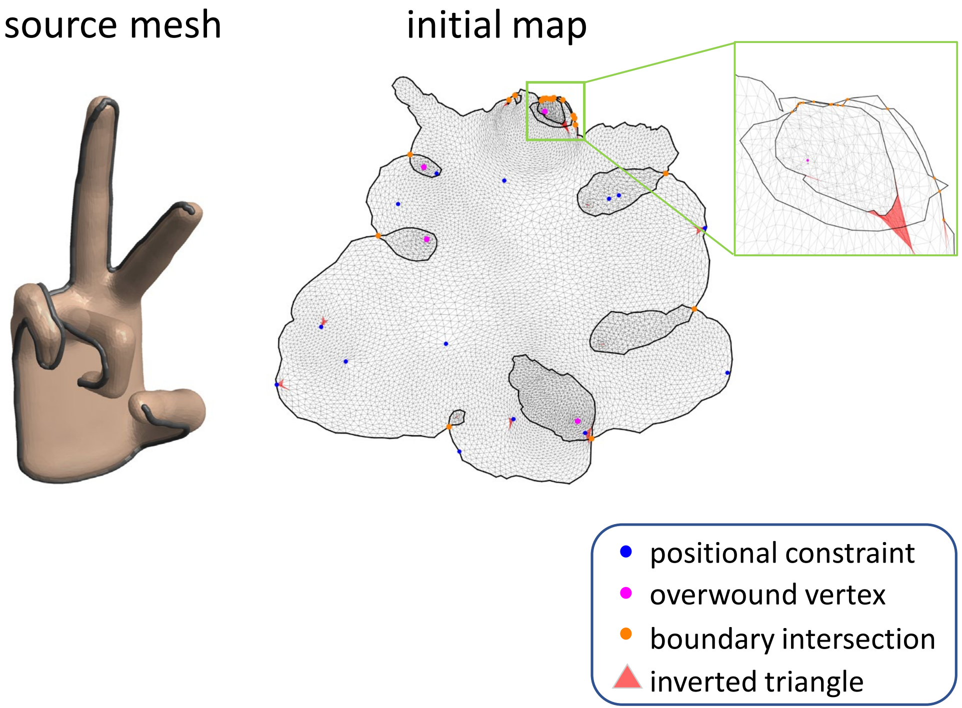
Large-scale Bounded
Distortion (LBD)
[Kovalsky et al. 2016]



Simplex
Assembly (SA)
[Fu and Liu 2016]

Benchmark

- 1791 test examples
  - Up to 20 constraints
  - Non-injective initial map by ARAP
- Parameters
  - $\alpha = 10^{-4}, \theta = 0.1$
- Comparison with inversion-free methods
  - LBD [Kovalsky et al. 2016]
  - SA [Fu and Liu 2016]



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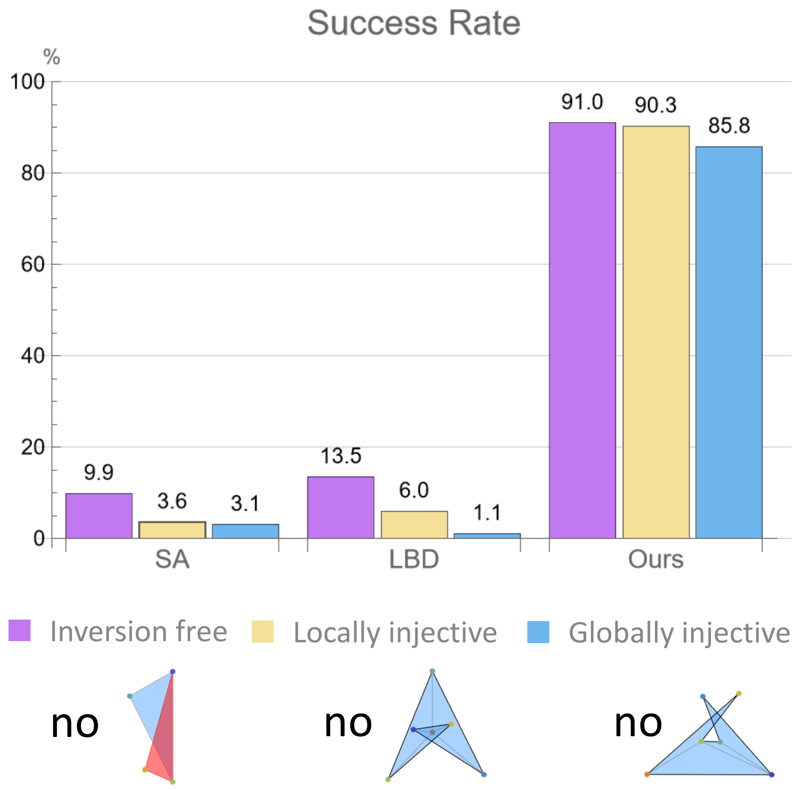
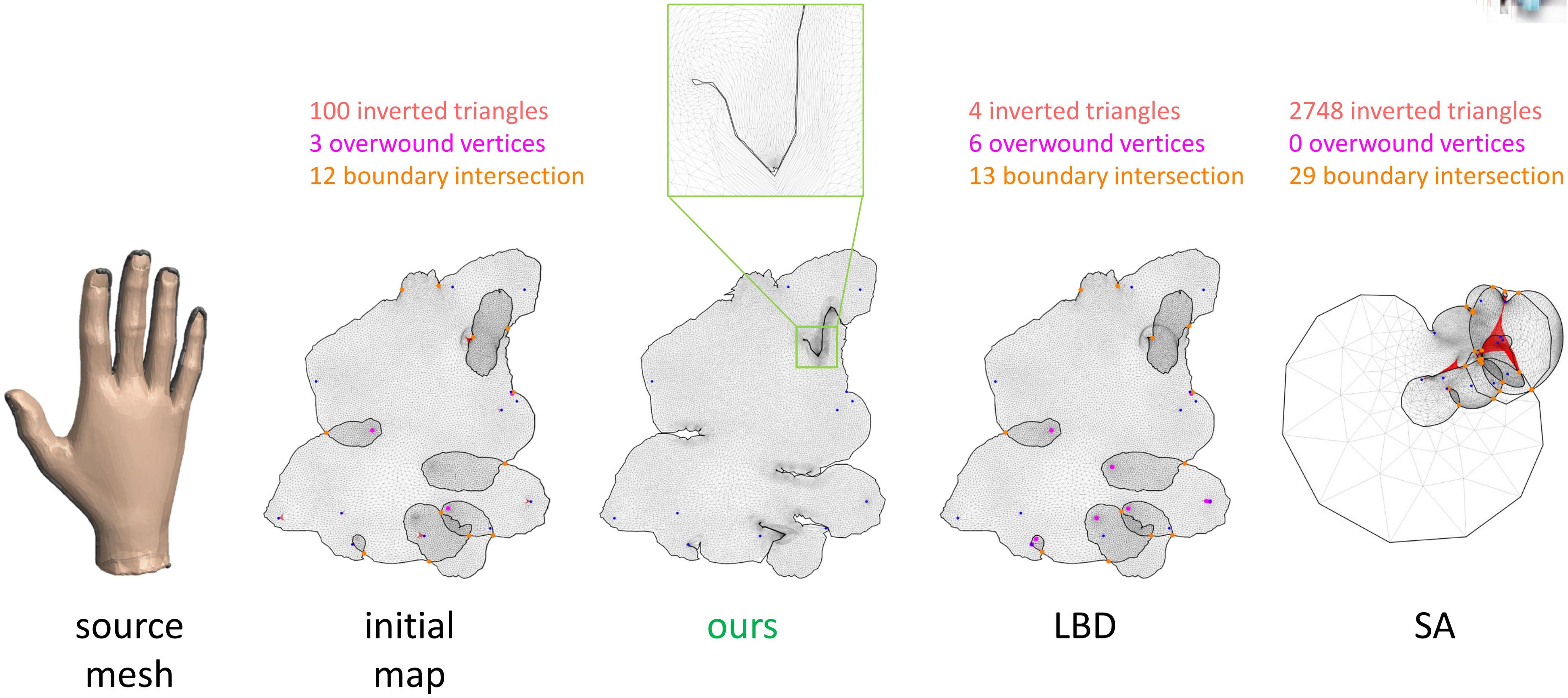
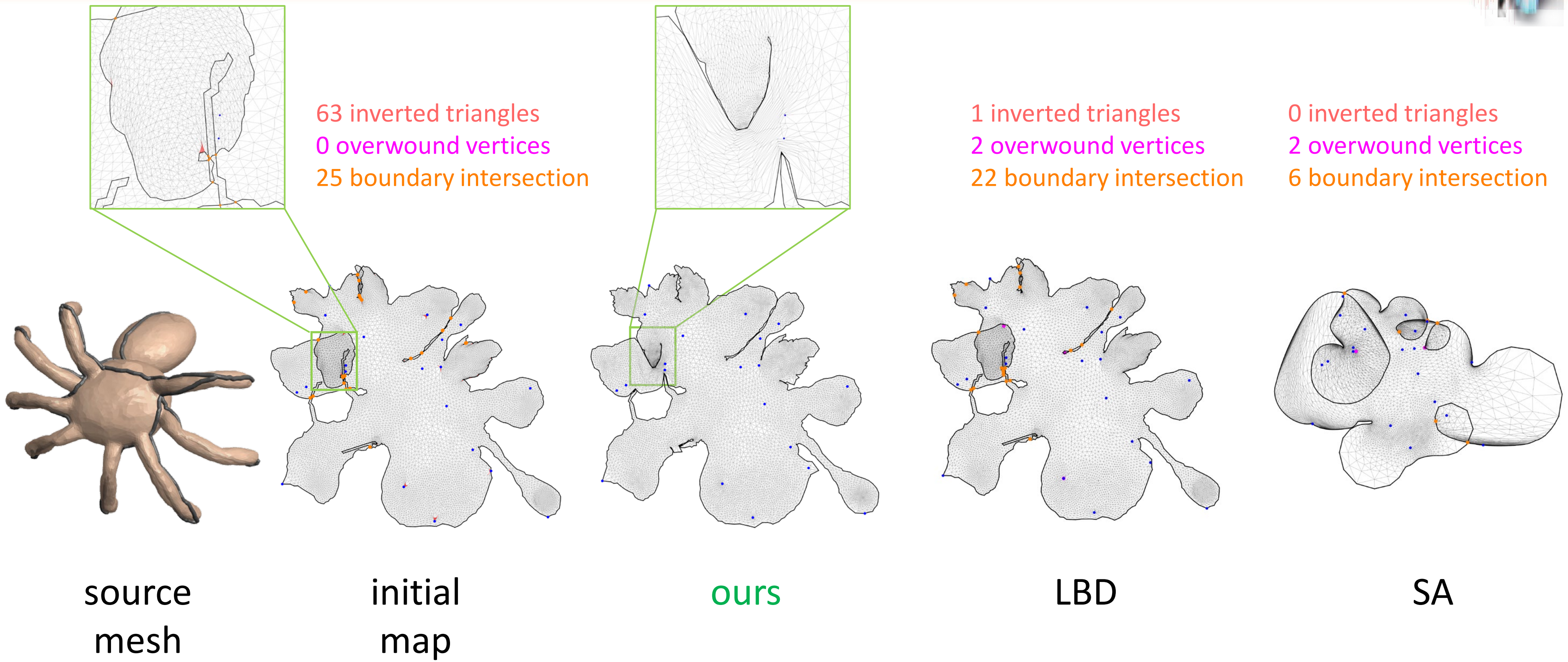


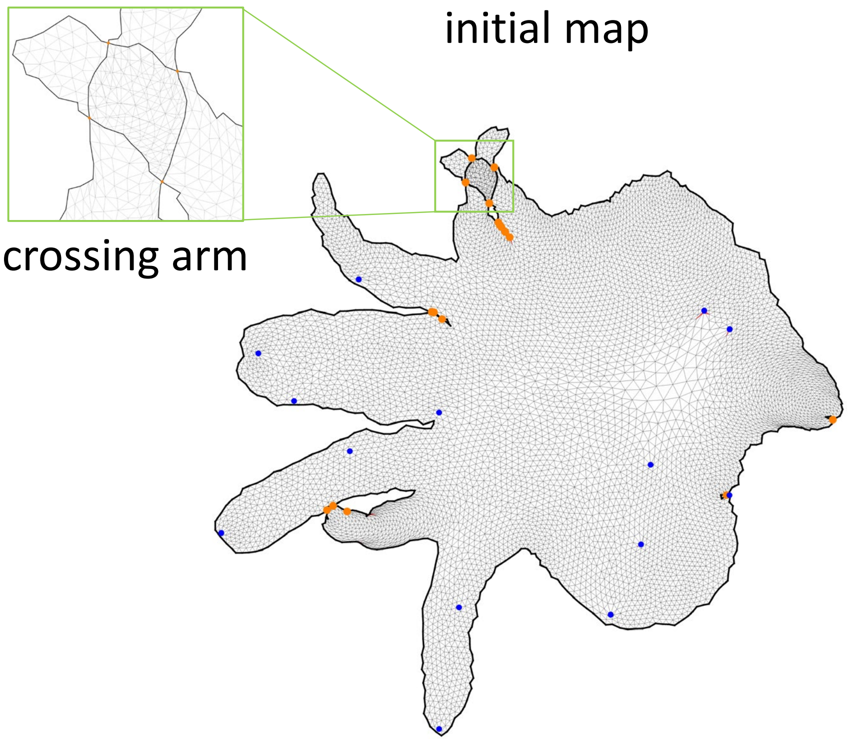
Benchmark: Success

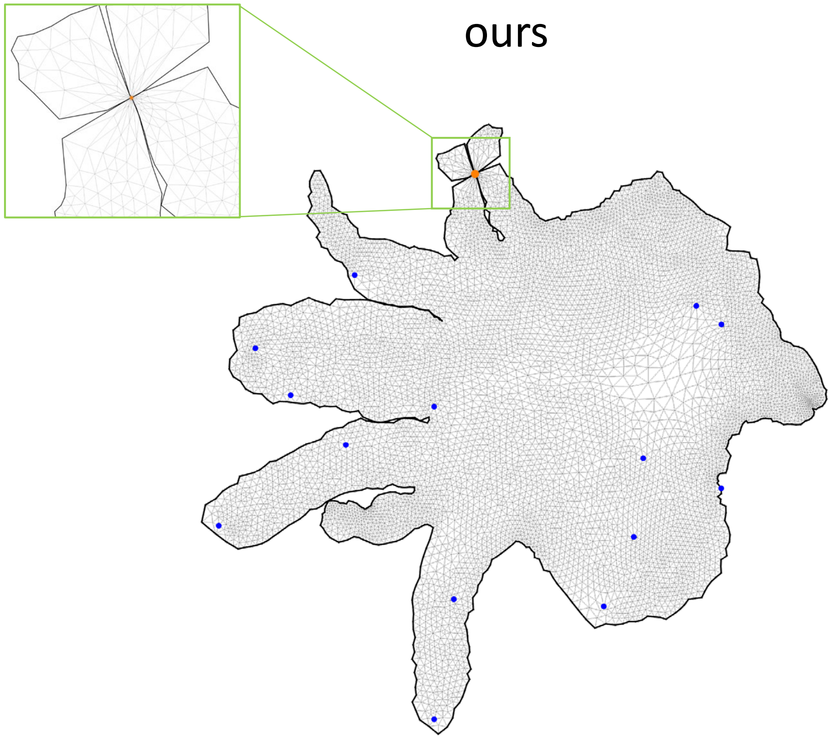


Benchmark: Success



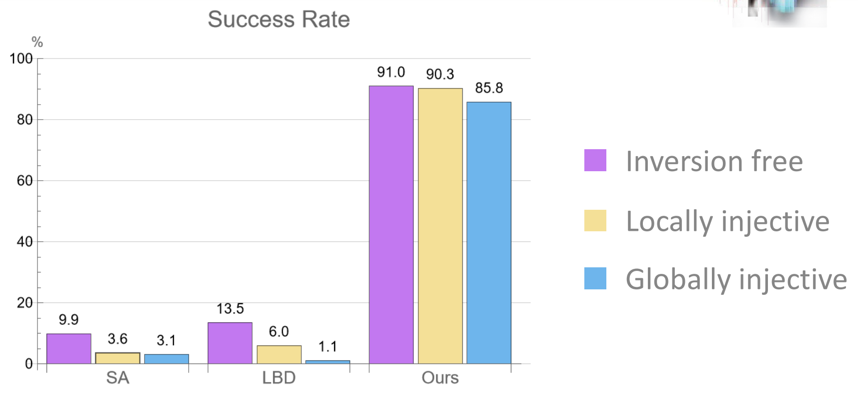
Benchmark: failure

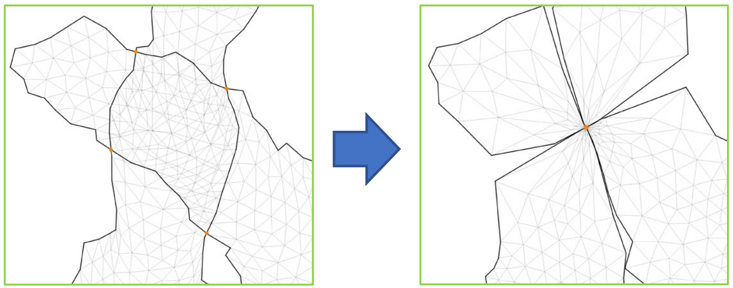




Summary

- First method to recover global injectivity under positional constraints
  - Significantly outperforms inversion-free methods
- Smooth Excess Area (SEA)
  - Captures both inverted and overlapping areas
  - Smooth almost everywhere
  - Theoretical guarantees of injectivity at global minima
- Future directions
  - Improve convergence rate
  - Higher order smoothness
  - Extend to 3D





crossing arm

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