SIGGRAPH THINK BEYOND

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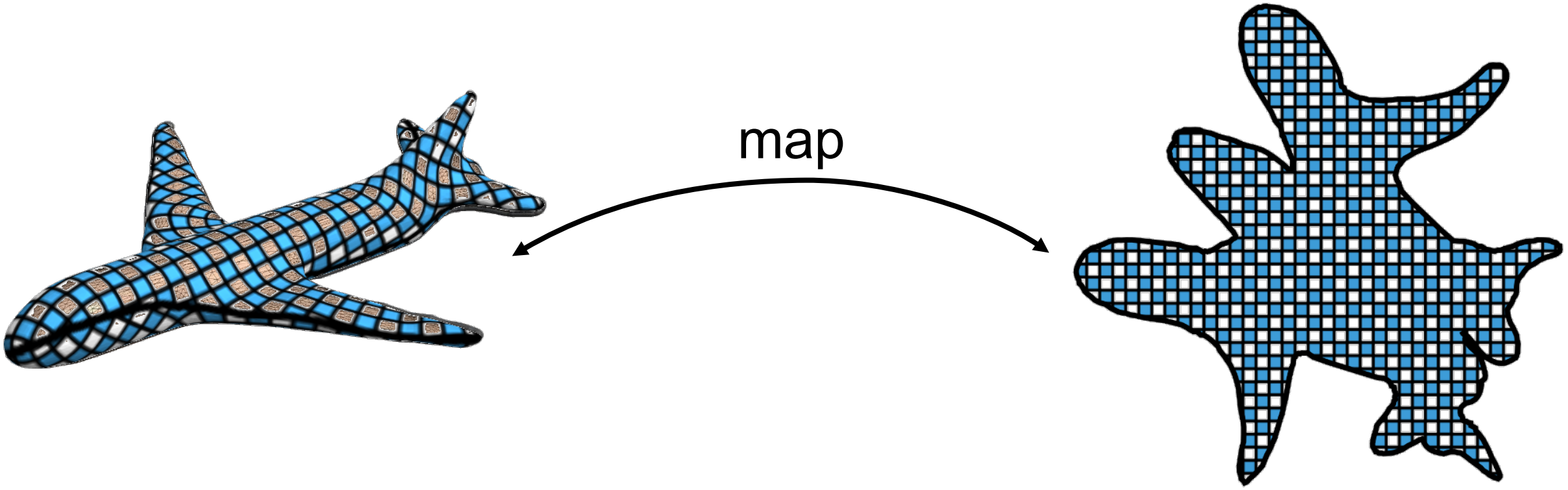
LIFTING SIMPLICES TO FIND INJECTIVITY

XINGYI DU, Washington University in St. Louis, USA
NOAM AIGERMAN and QINGNAN ZHOU, Adobe Research, USA
SHAHAR Z. KOVALSKY, Duke University, USA
YAJIE YAN, Facebook, USA
DANNY M. KAUFMAN, Adobe Research, USA
TAO JU, Washington University in St. Louis, USA

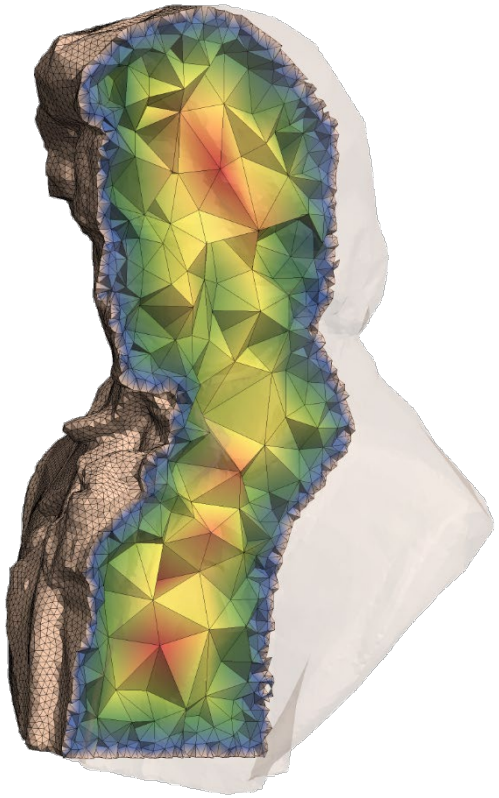
MAPPING APPLICATIONS

Texture mapping

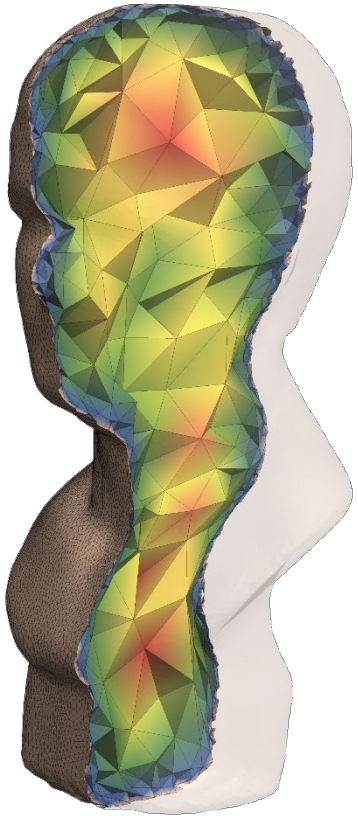


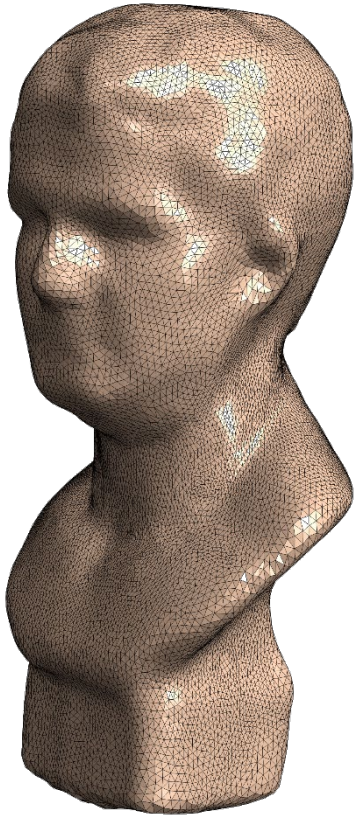
MAPPING APPLICATIONS

Data transfer



map





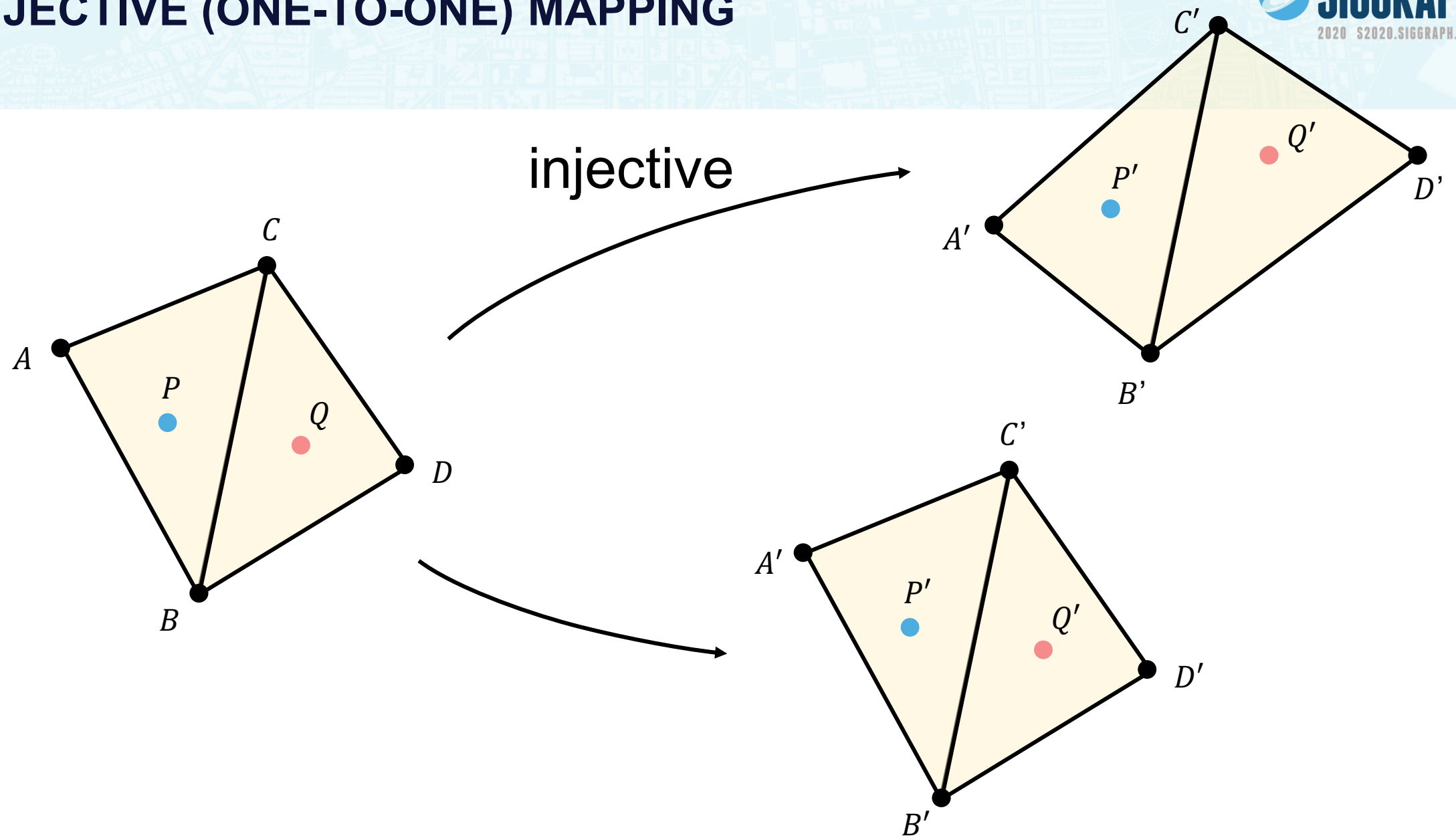
source surface

Data

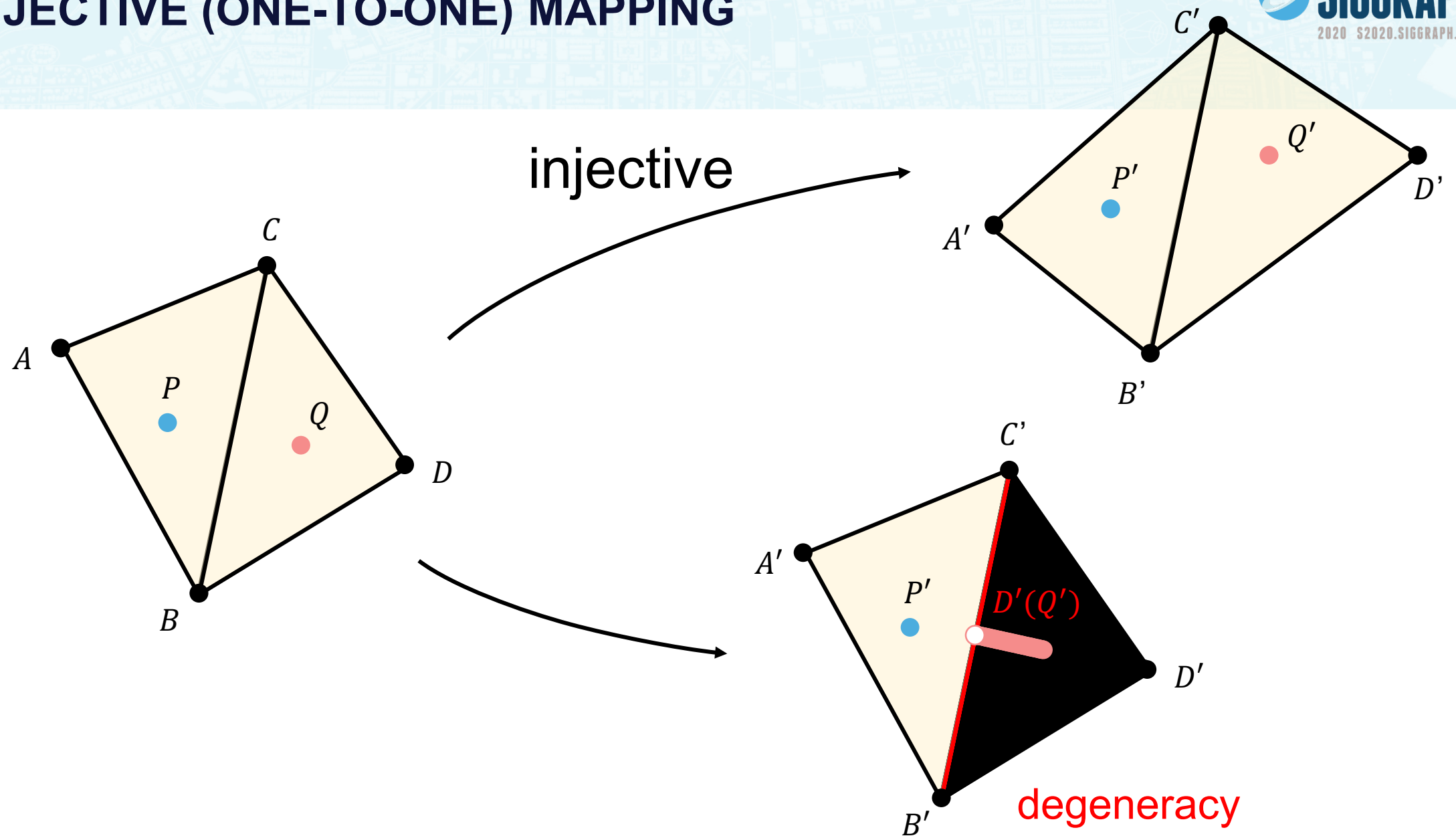
target domain

target surface

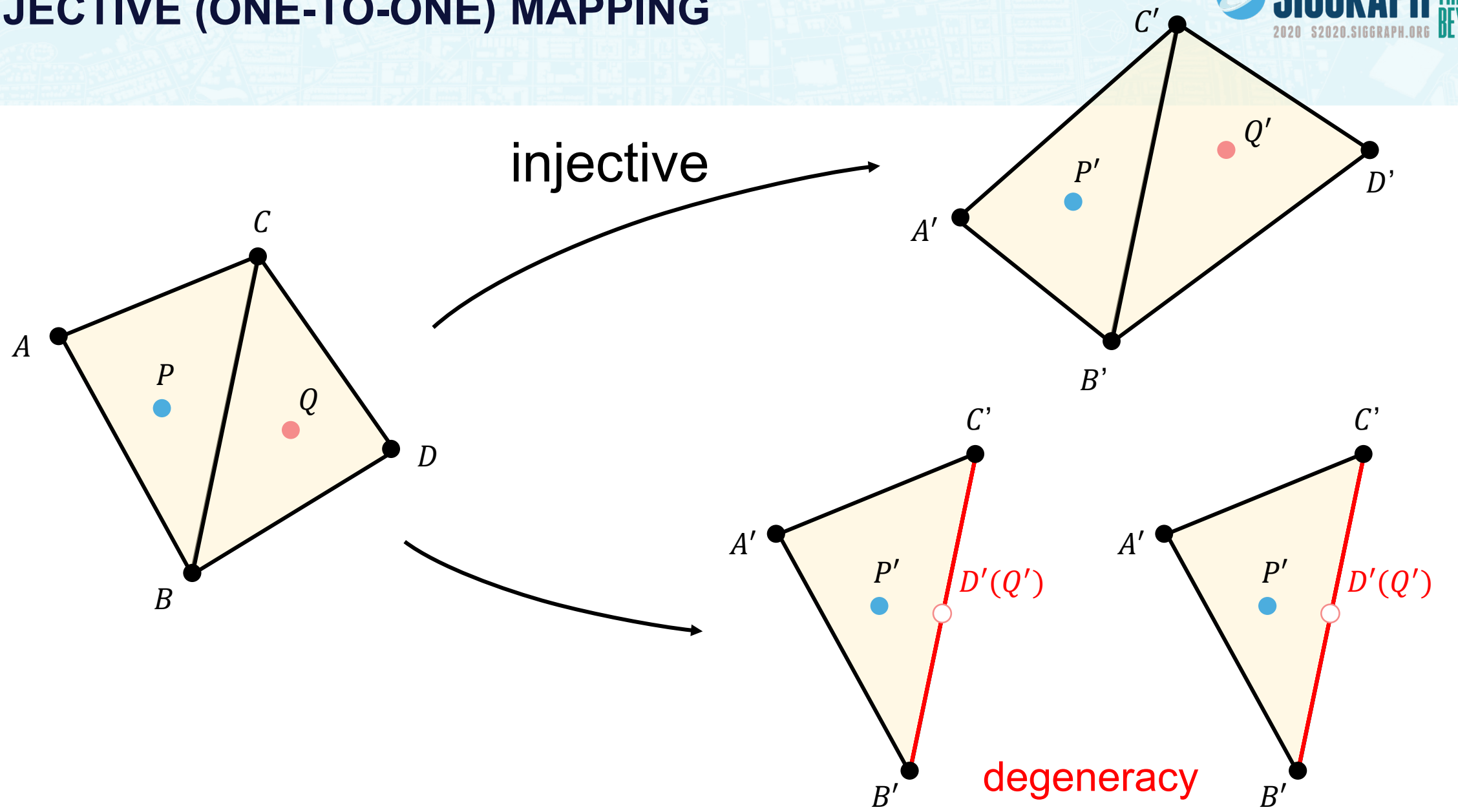
INJECTIVE (ONE-TO-ONE) MAPPING



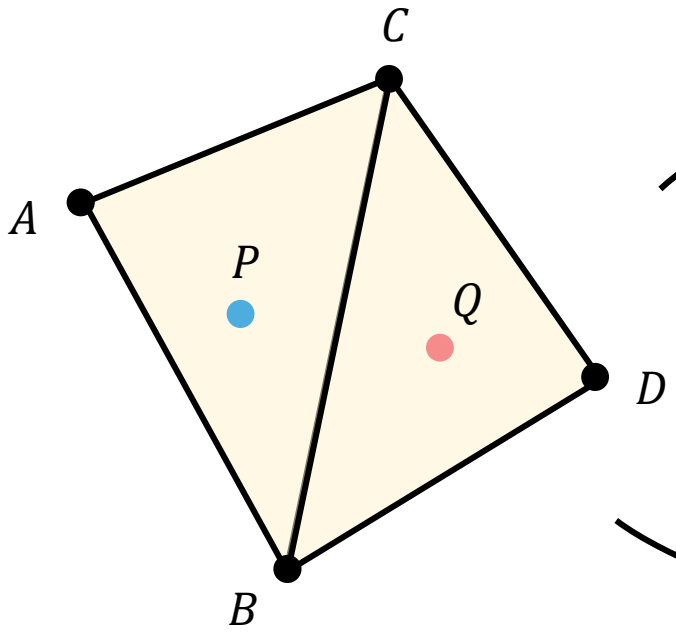
INJECTIVE (ONE-TO-ONE) MAPPING



INJECTIVE (ONE-TO-ONE) MAPPING

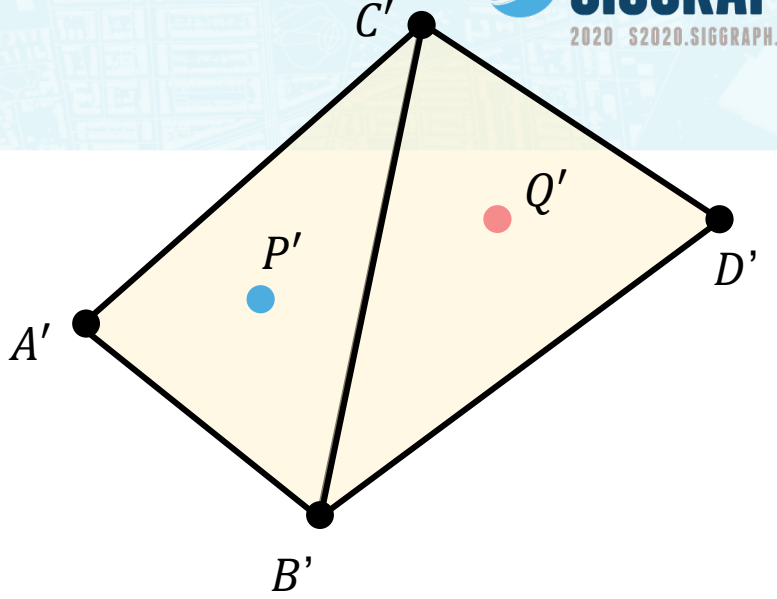


INJECTIVE (ONE-TO-ONE) MAPPING

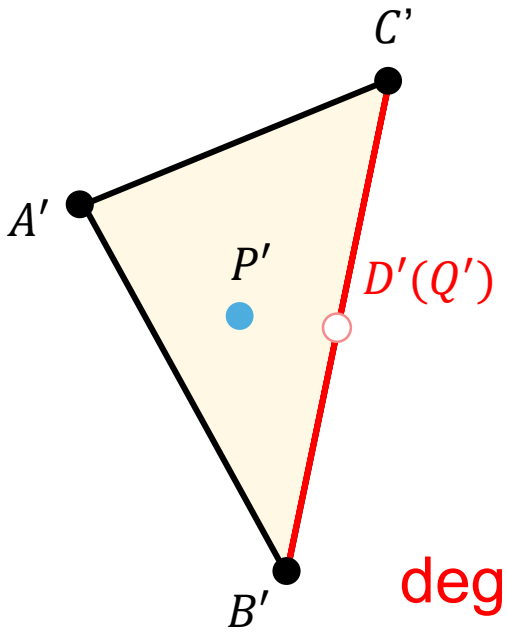


injective

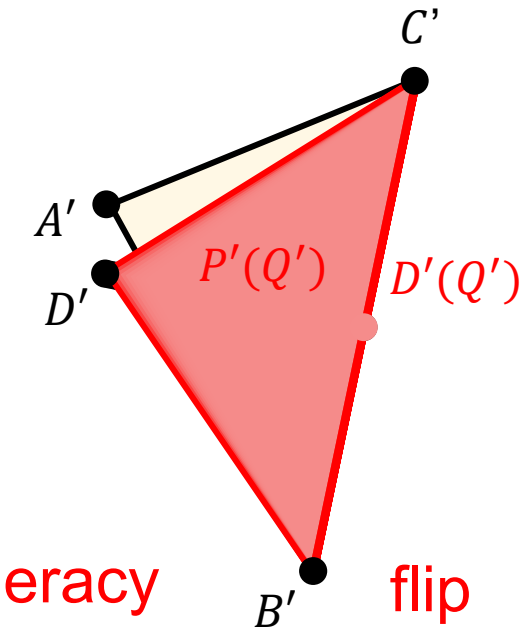
no flip/degeneracy



non-injective



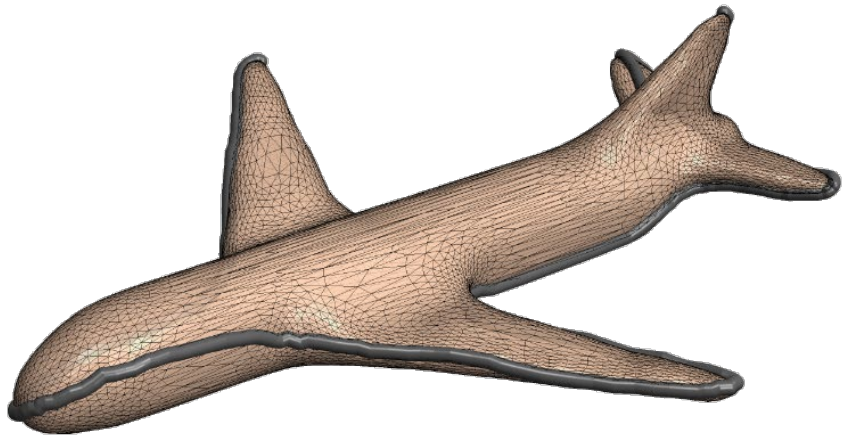
degeneracy



flip

FIXED-BOUNDARY INJECTIVE MAPPINGS

input: (1) source mesh



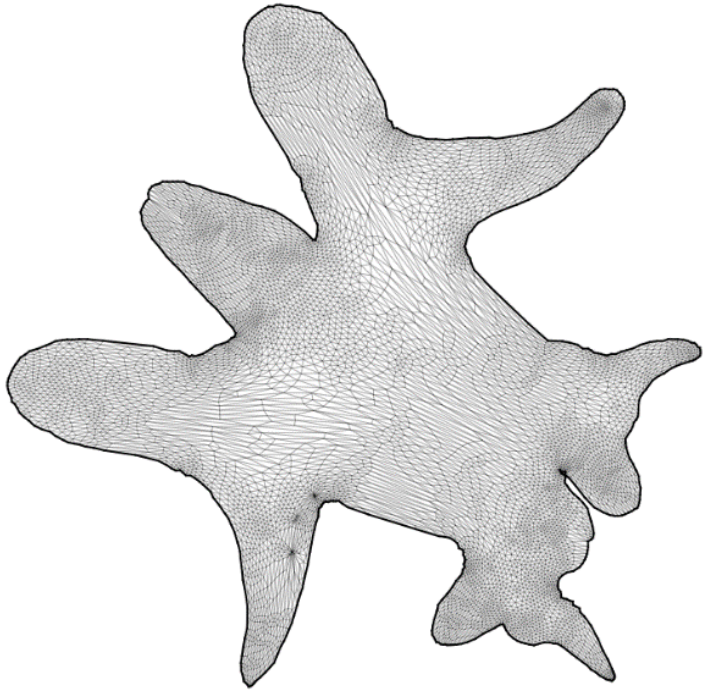
output: mapping

no flip/degeneracy

injective

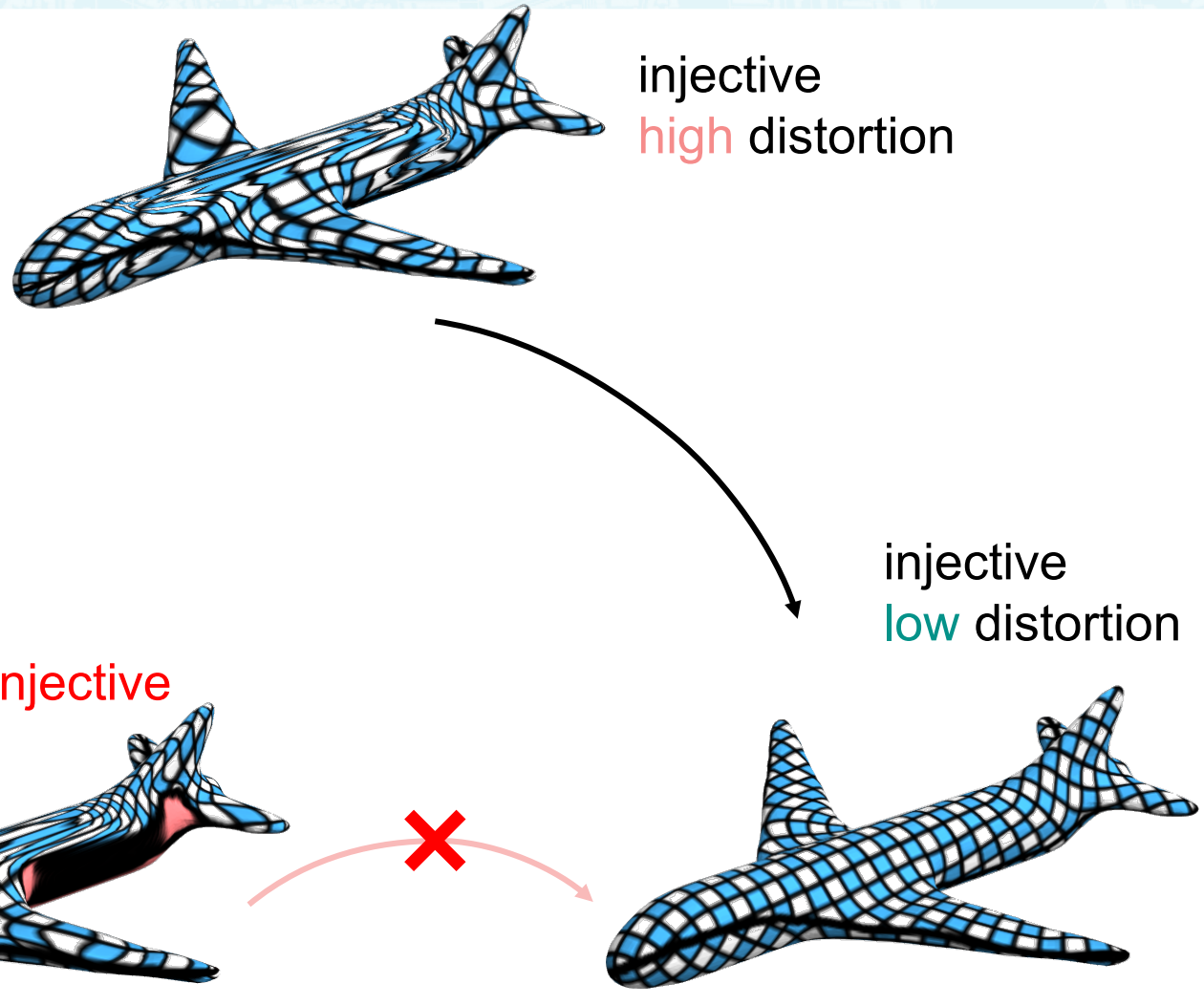
(2) target boundary

- correspond to source boundary
- no self-intersection



PREVIOUS WORK

- Minimize mapping distortion
  - [Schüller et al. 2013], [Liu et al. 2016], [Smith and Schaefer 2015], AMIPS [Fu et al. 2015], SLIM [Rabinovich et al. 2017], CM [Shtengel et al. 2017], [Claici et al. 2017], BCQN [Zhu et al. 2018], [Liu et al. 2018]
  - require injective mappings as initialization



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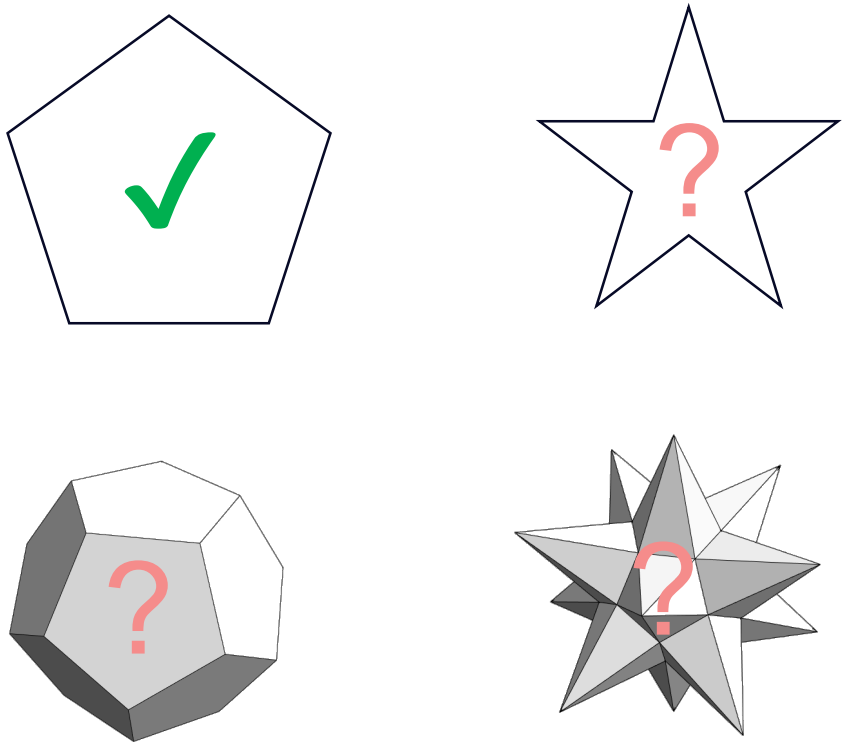
- Tutte Embedding [Tutte 1963]
  - guarantee injectivity for 2D convex domains
  - no guarantee for non-convex or 3D domains

2D Convex Domain



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  - require injective mappings as initialization
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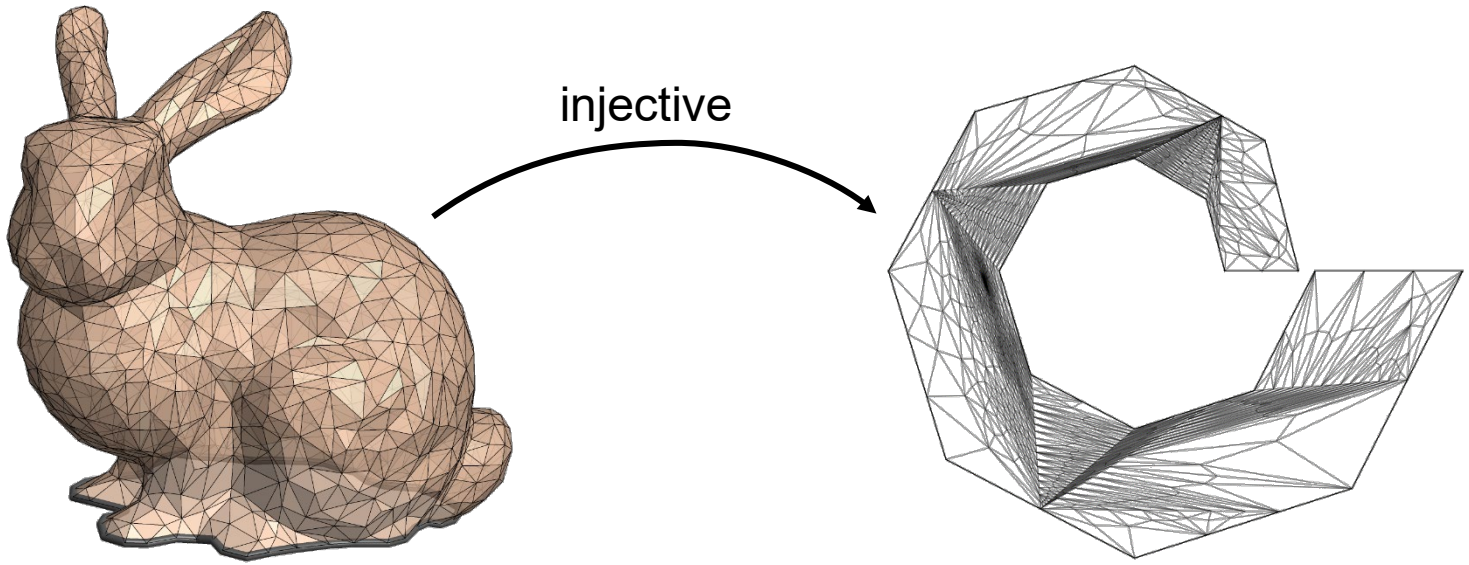
  - no guarantee for non-convex or 3D domains
- Produce injective mappings
  - may change mesh structure [Agarwal et al. 2008], [Weber and Zorin 2014], [Campen et al. 2016], [Gu et al. 2018], [Shen et al. 2019]



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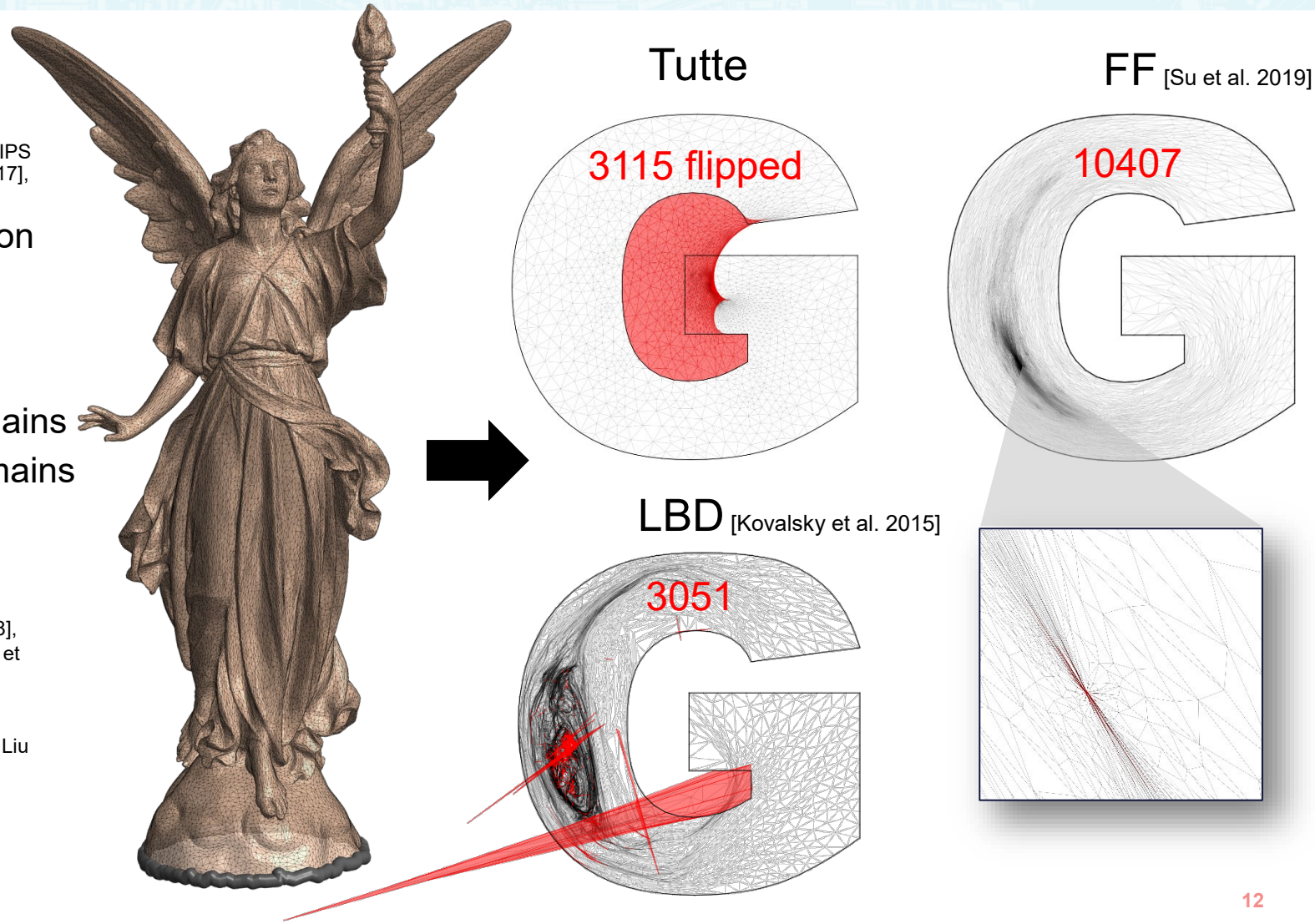
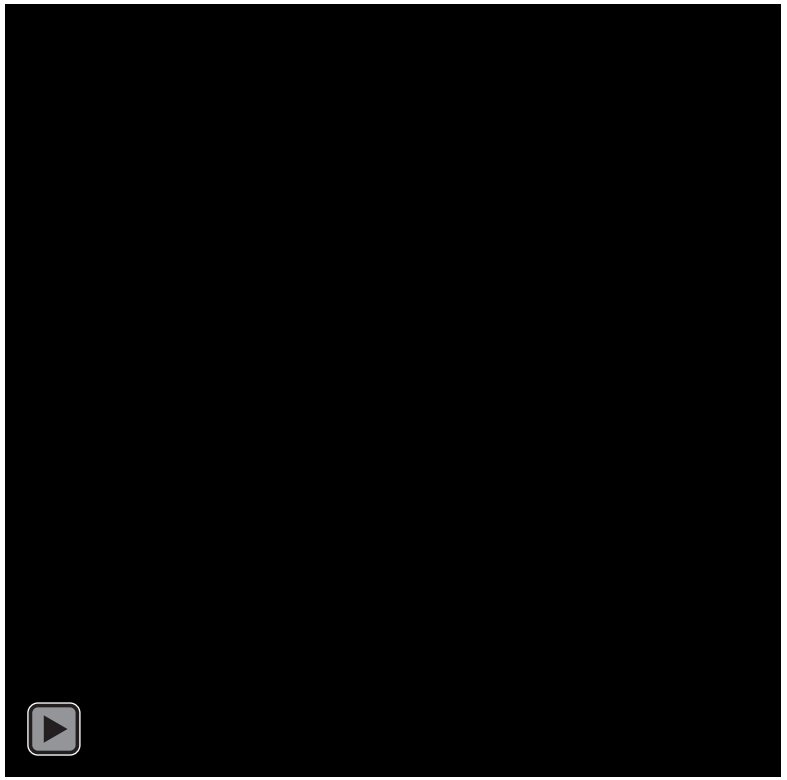
- Produce injective mappings
  - may change mesh structure [Agarwal et al. 2008], [Weber and Zorin 2014], [Campen et al. 2016], [Gu et al. 2018], [Shen et al. 2019]
  - often fail on complex target domains [Aigerman and Lipman 2013], **LBD** [Kovalsky et al. 2015], **SA** [Fu and Liu 2016], **FF** [Su et al. 2019]



CONTRIBUTION

- New method to produce injective mappings
  - fixed-boundary domain in 2D/3D
  - maintain mesh structure
- New energy (Total Lifted Content, TLC)
  - theory: global minima are injective
  - practice: high success rate



CONTRIBUTION

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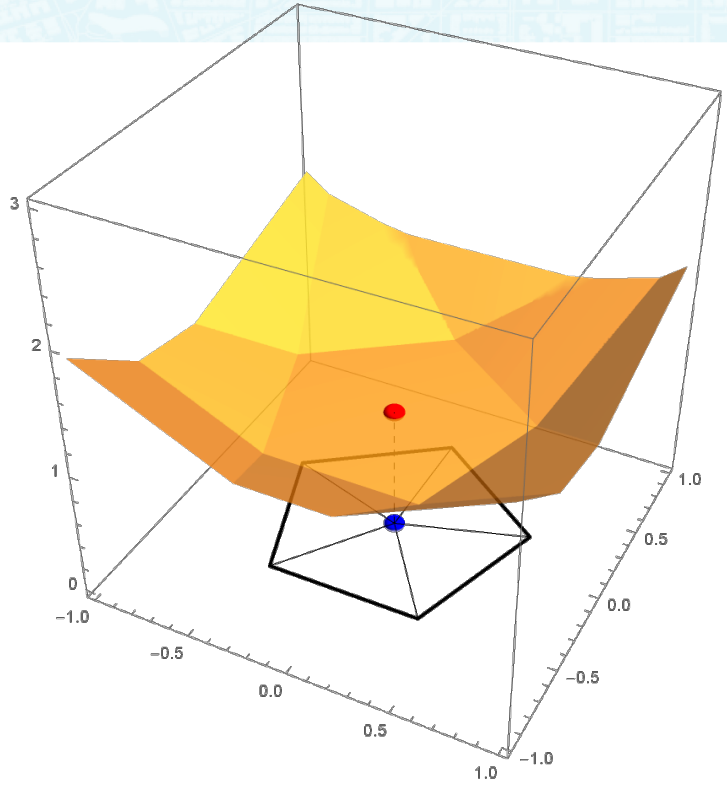
  - maintain mesh structure
- New energy (Total Lifted Content, TLC)
  - theory: global minima are injective
  - practice: high success rate
- Benchmark dataset

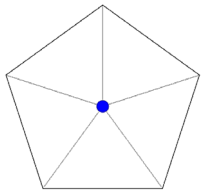
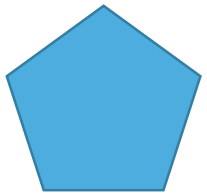
10734 triangle meshes
904 tetrahedron meshes

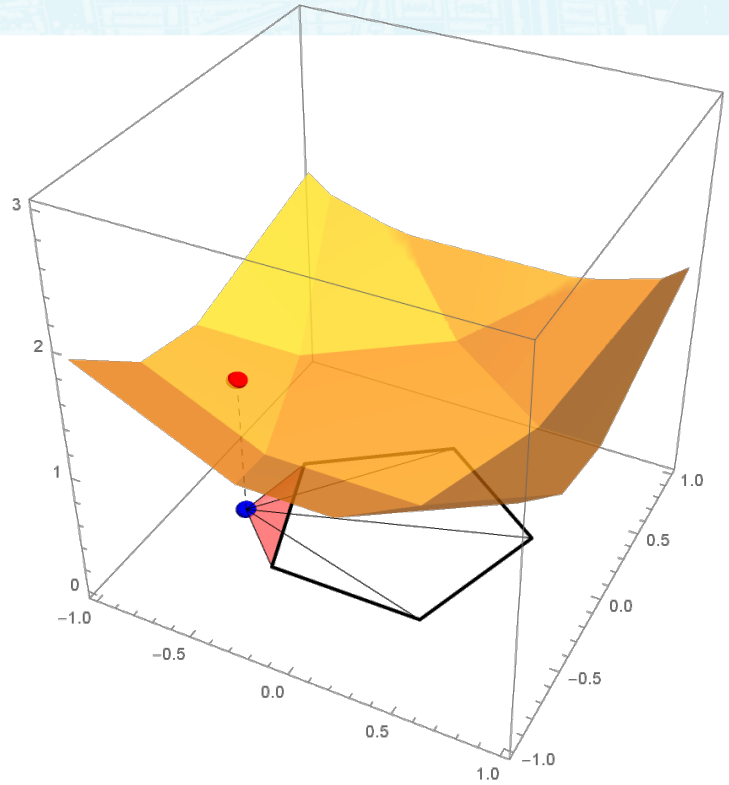
TOTAL LIFTED CONTENT

TOTAL UNSIGNED AREA (TUA) [XU ET AL. 2011]

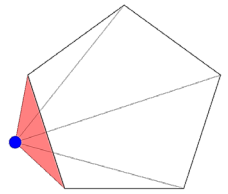
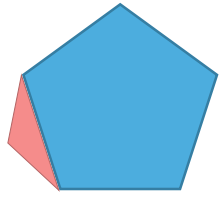


no flip

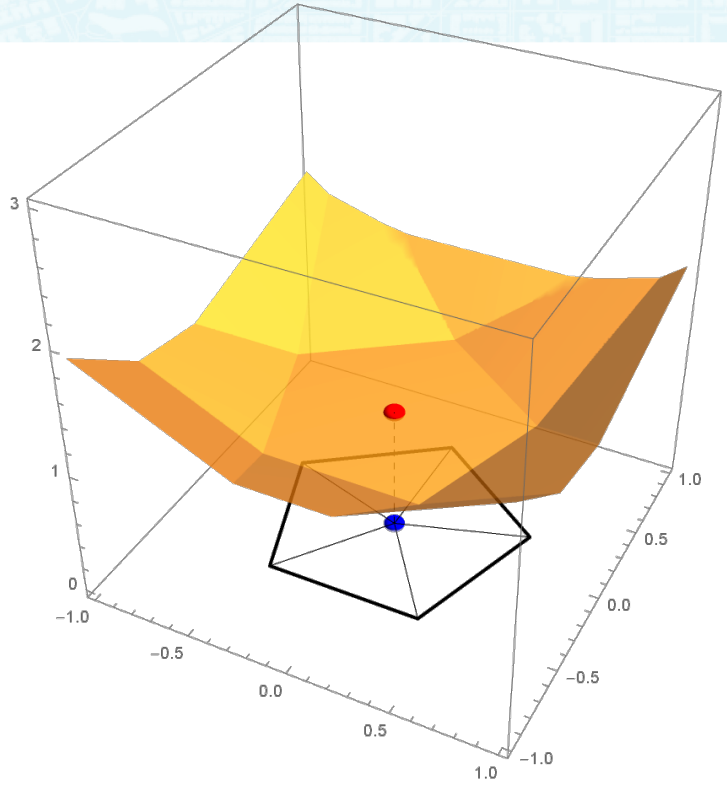
total area () = 



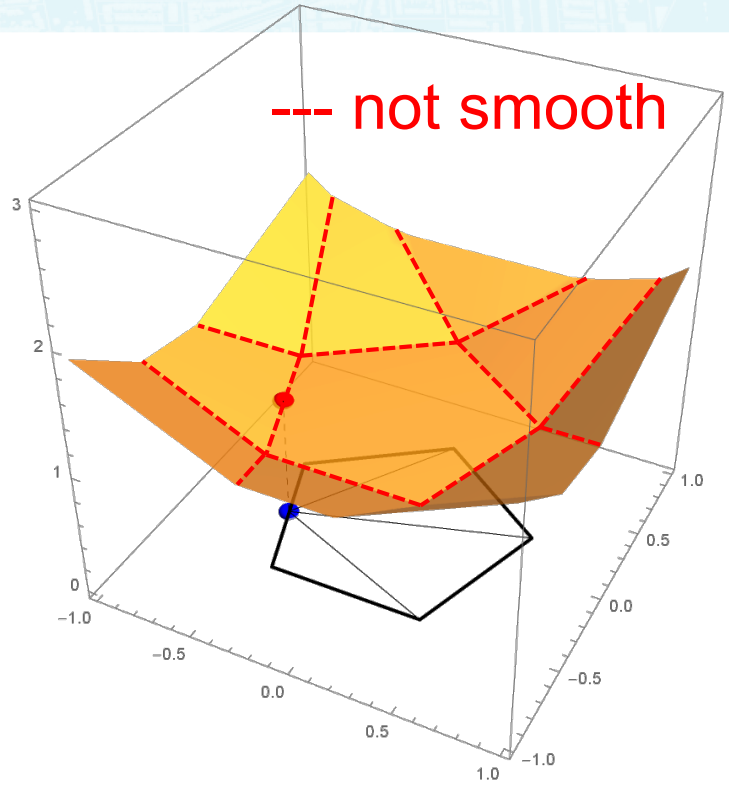
flip

total area () =  + 

PROBLEM OF TUA [XU ET AL. 2011]

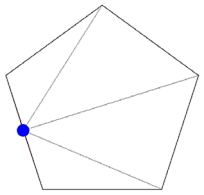


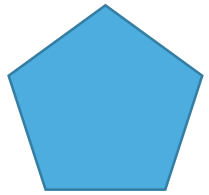
global minimum ⇒ no flip



non-injective global minimum

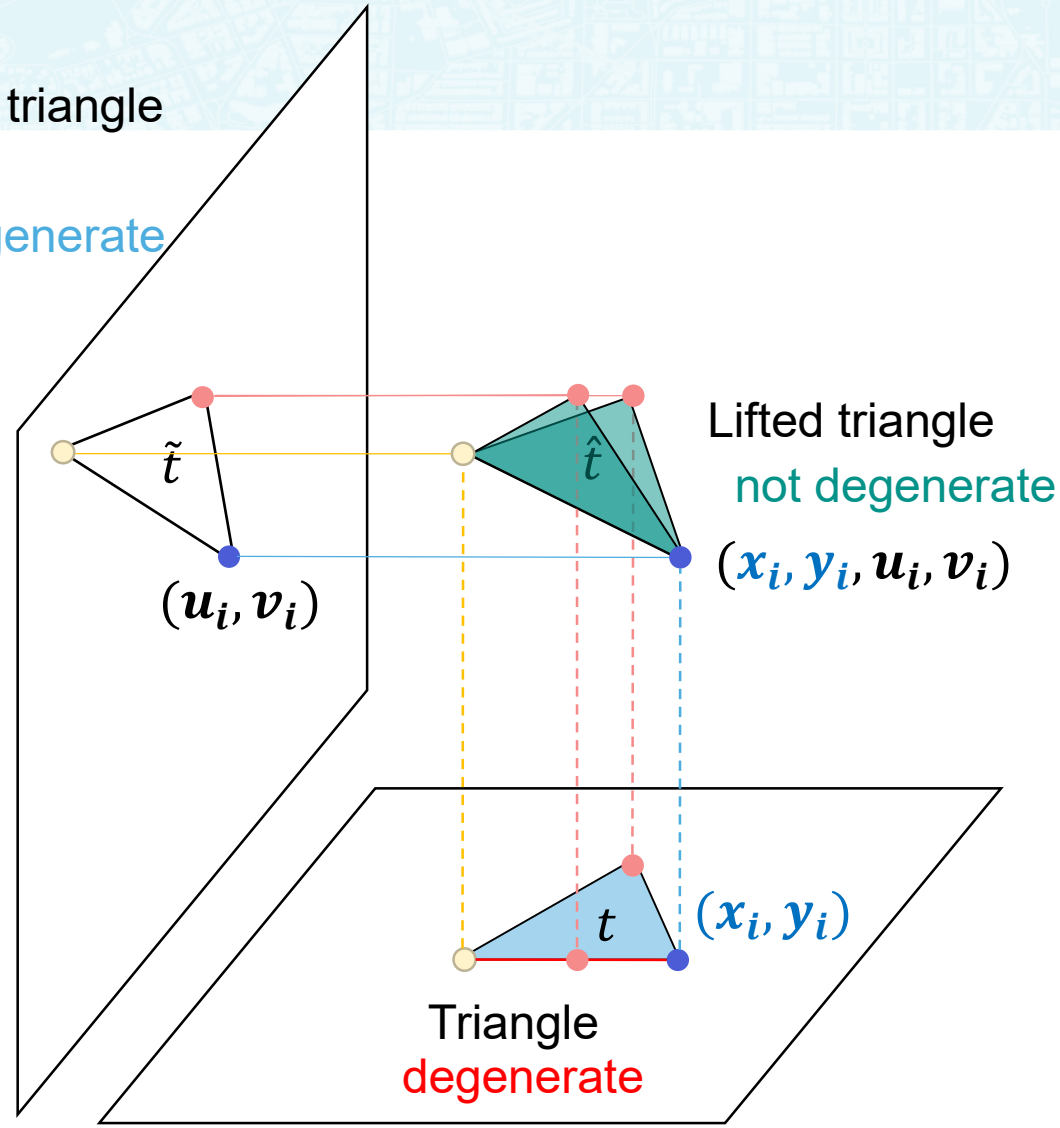
total area ( ) =





Auxiliary triangle

- fixed
- not degenerate



--- not smooth

no degeneracy

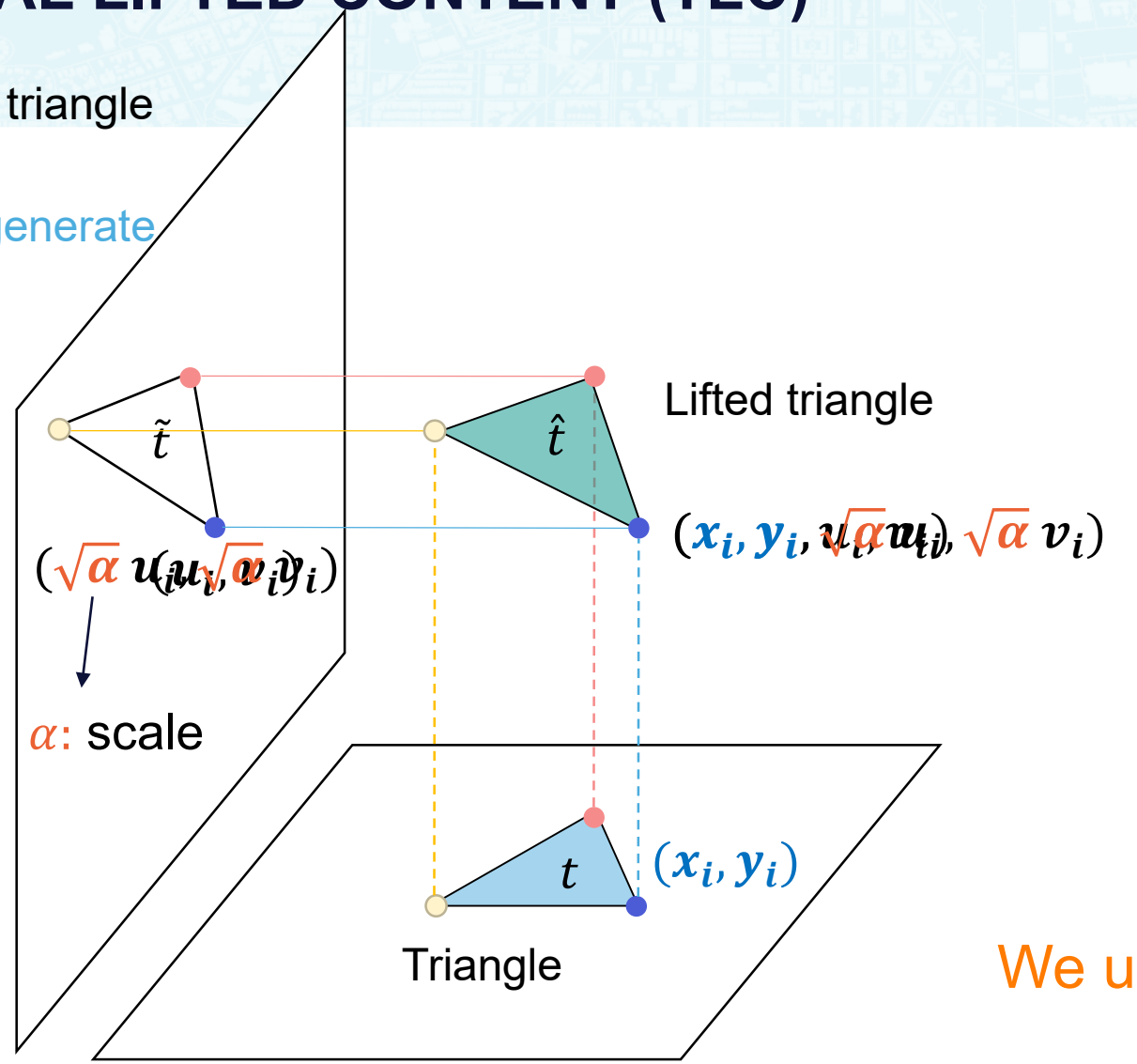
Lifting

degeneracy

non-injective global minimum

total area ( ) =

TOTAL LIFTED CONTENT (TLC)



Lifted content

$$\text{LiftedContent}(t) = \text{Area}(\hat{t})$$ triangle

$$\text{LiftedContent}(t) = \text{Volume}(\hat{t})$$ tetrahedron

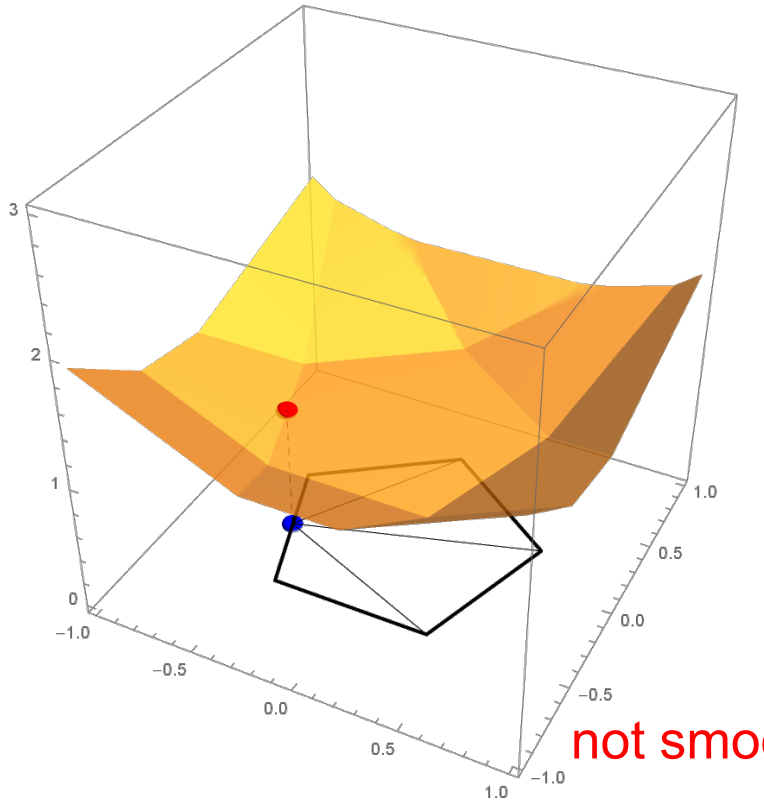
Total lifted content of a mesh

$$\text{TLC}(Mesh) = \sum_{t \in Mesh} \text{LiftedContent}(t)$$

We use equilateral auxiliary simplices

TUA VS TLC

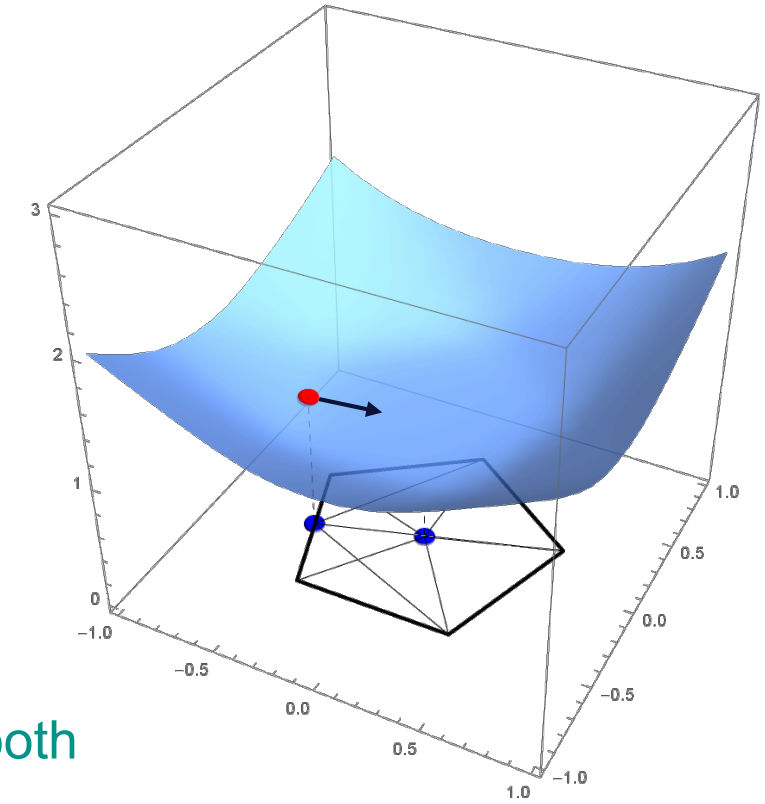
Total Unsigned Area



not smooth

non-injective global minimum

Total Lifted Content

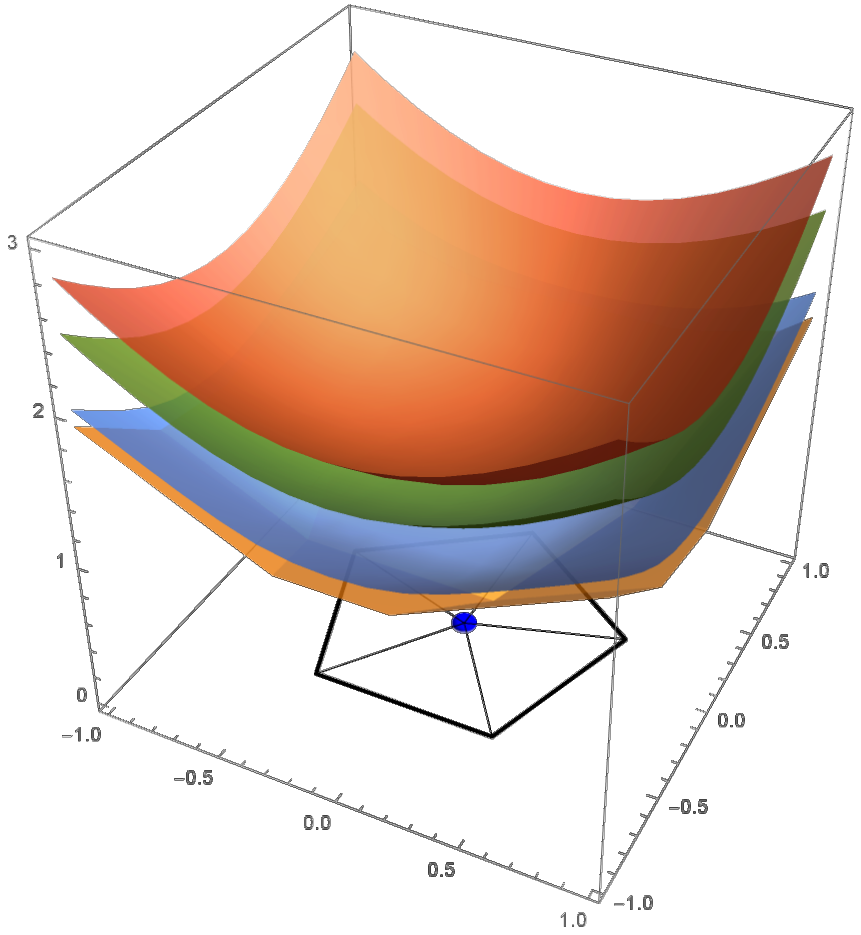


smooth

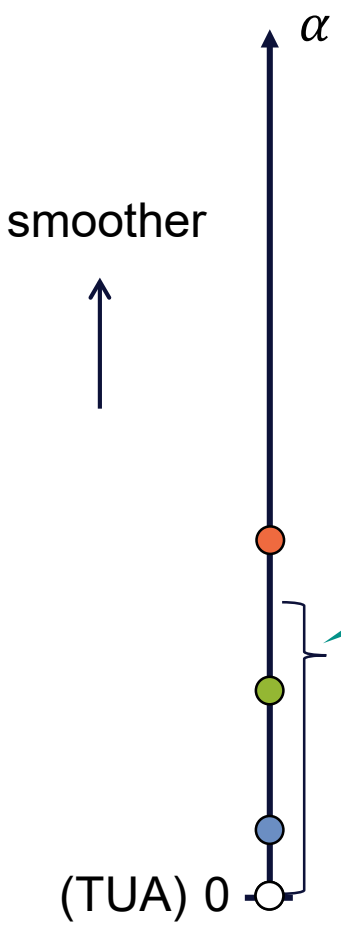
global minimum is injective

PARAMETER α

TLC with different α



smoother



$\alpha \to \infty$
- minimize Dirichlet energy (auxiliary → target)
- equilateral auxiliary simplices ⇒ Tutte embedding

TLC global minima are injective

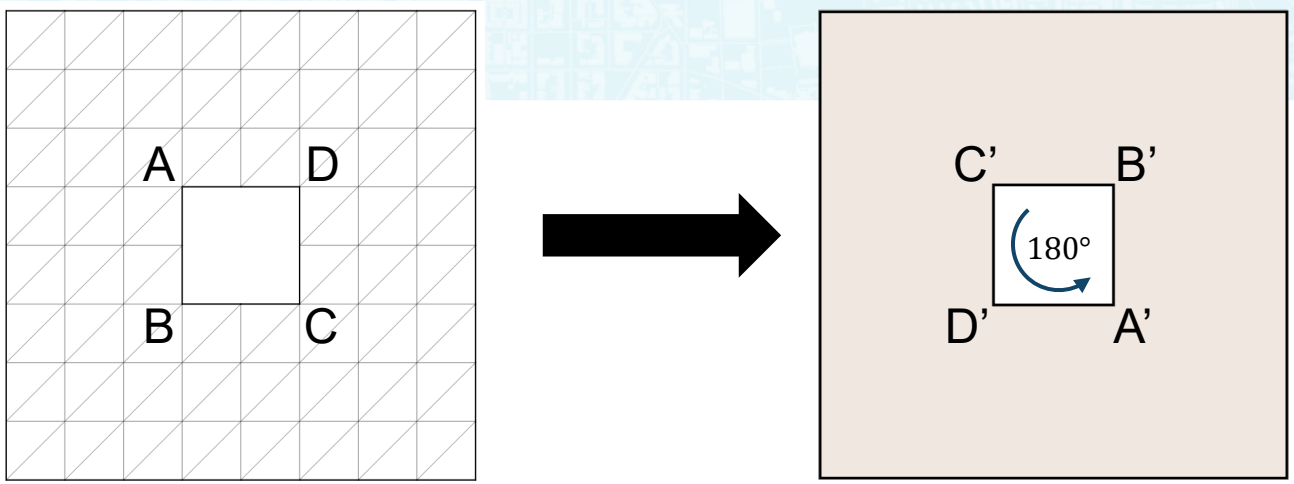
$\alpha \to 0$
- minimize Dirichlet energy (target → auxiliary)
- 2D: MIPS energy

[Hormann and Greiner, 2000]

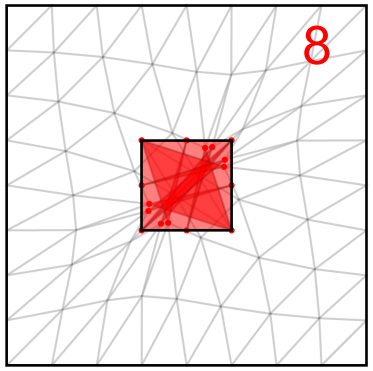
BENCHMARK

[Aigerman and Lipman 2013]
[Weber and Zorin 2014]
[Fu et al. 2016]
Scaffold [Jiang et al. 2017]
[Liu et al. 2018]
FF [Su et al. 2019]
IPC [Li et al. 2020]

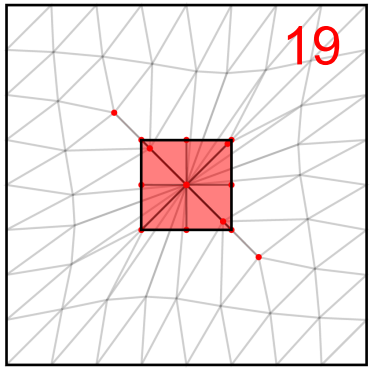
2D SIMPLE



flipped triangle

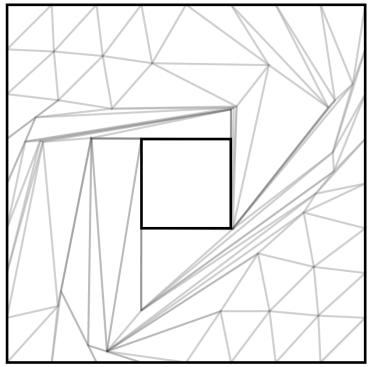


Tutte



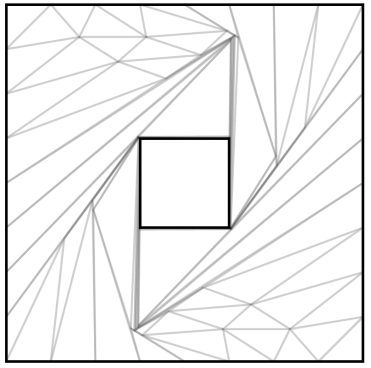
TUA

$(\alpha = 0)$



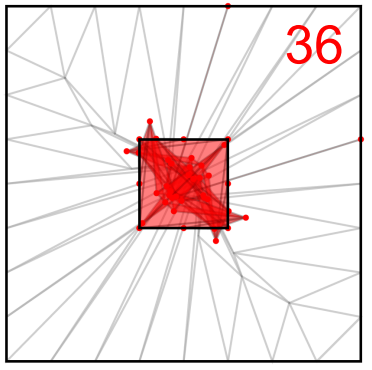
TLC

$(\alpha = 10^{-6})$



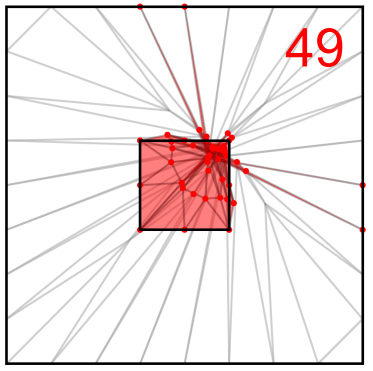
FF

[Su et al. 2019]



LBD

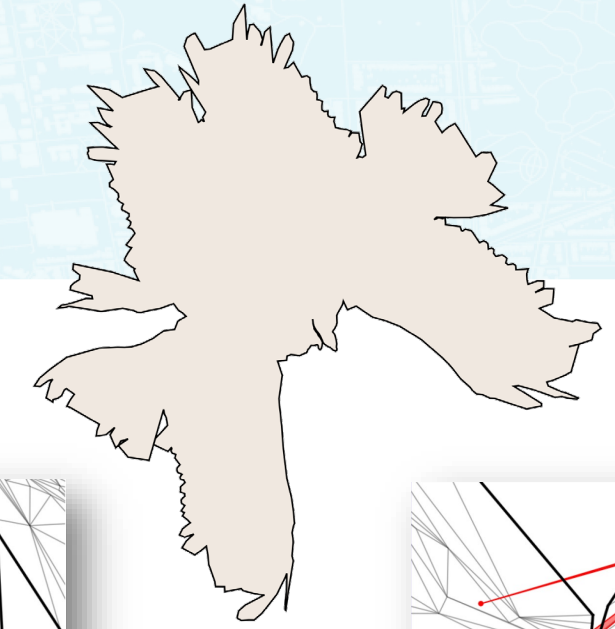
[Kovalsky et al. 2015]

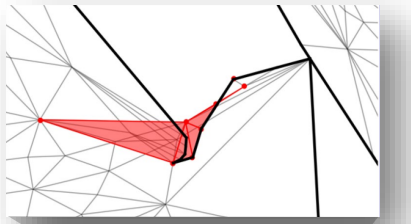


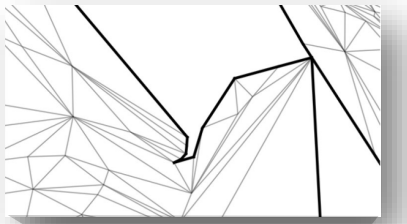
SA

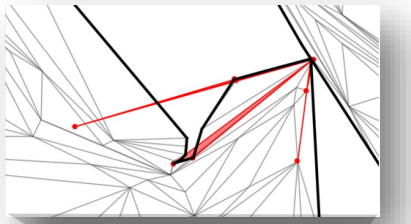
[Fu and Liu 2016]

2D COMPLEX





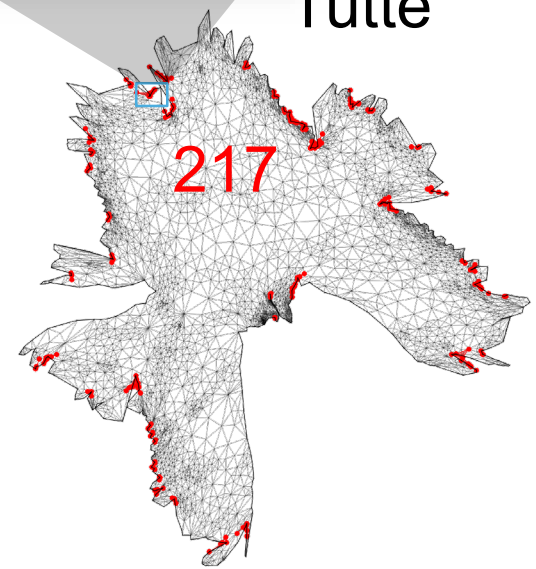


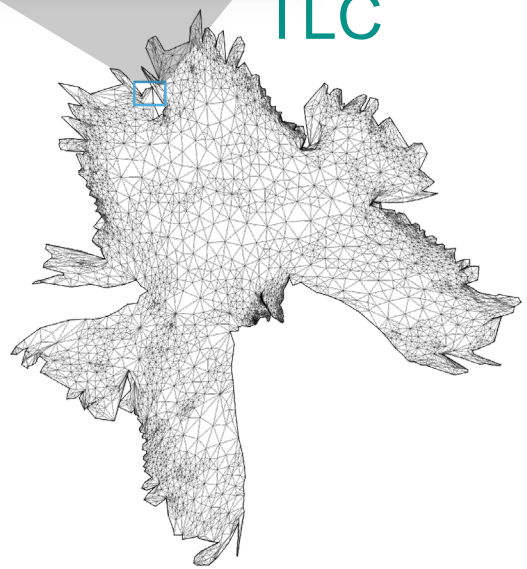


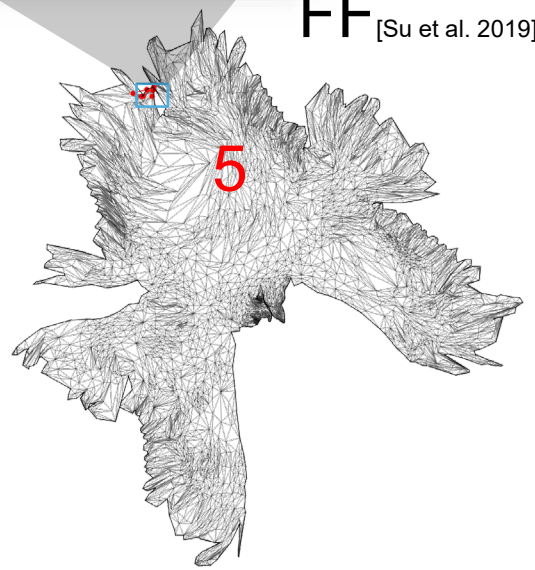
Tutte

TLC

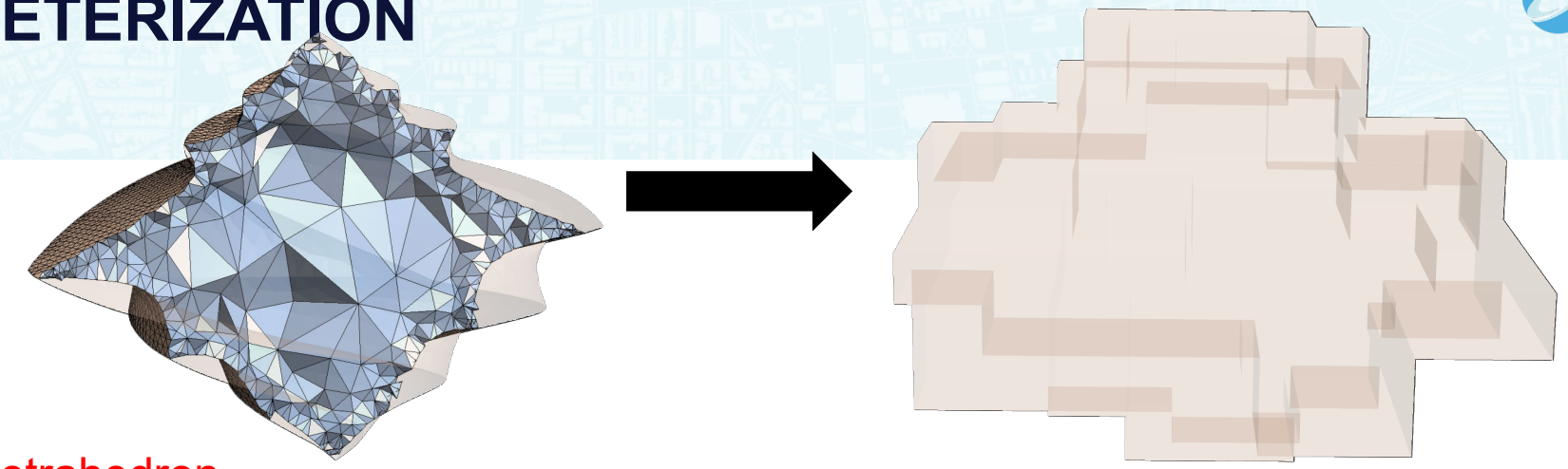
FF [Su et al. 2019]





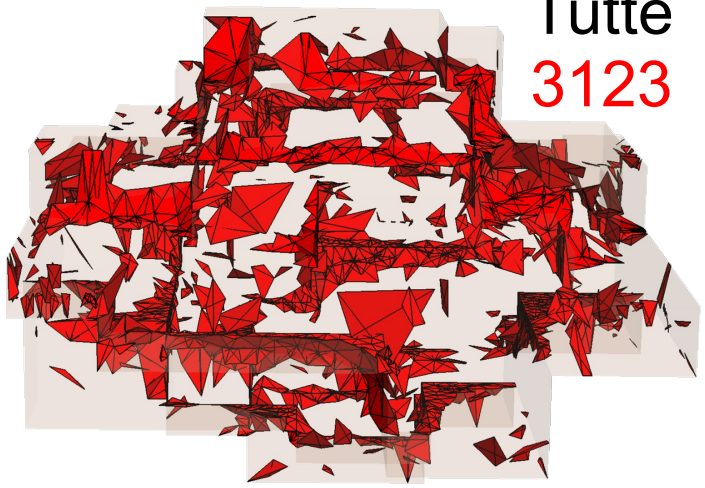


3D PARAMETERIZATION

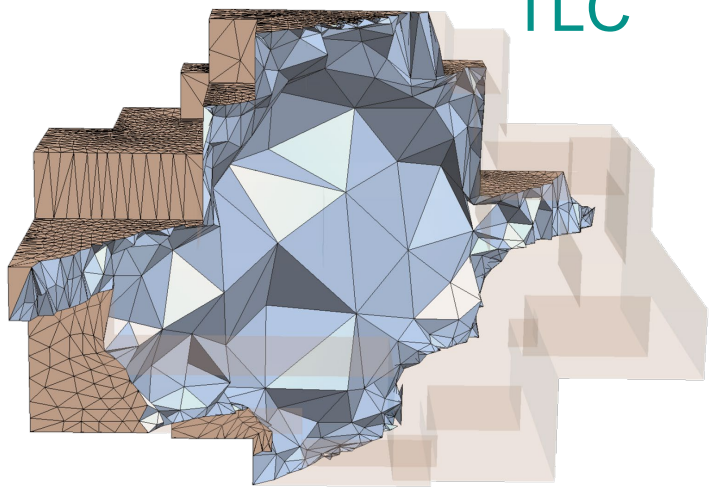


flipped tetrahedron

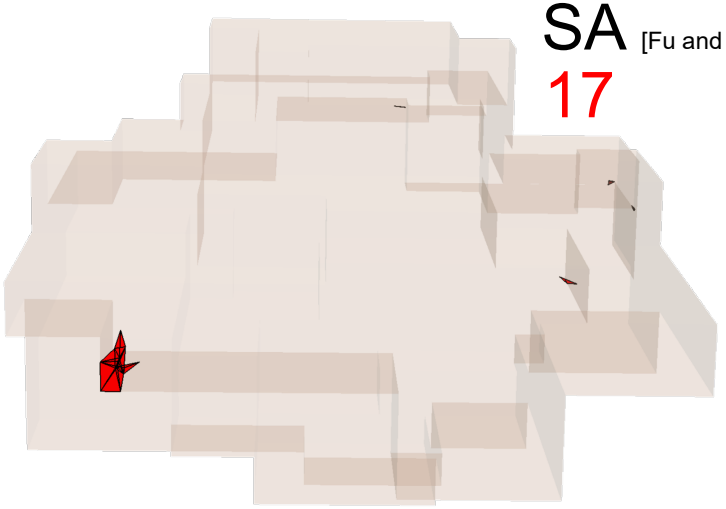
Tutte
3123



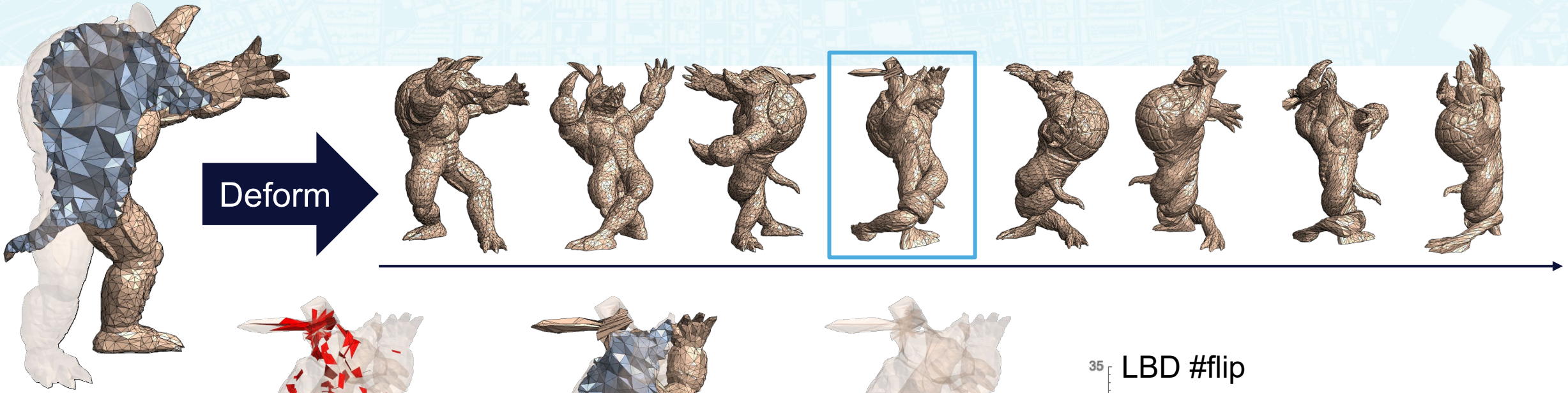
TLC

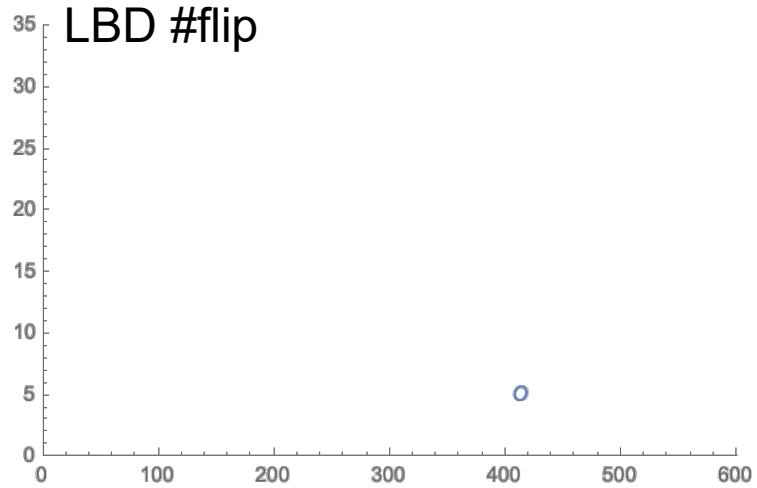


SA [Fu and Liu 2016]
17



3D DEFORMATION





Tutte

283

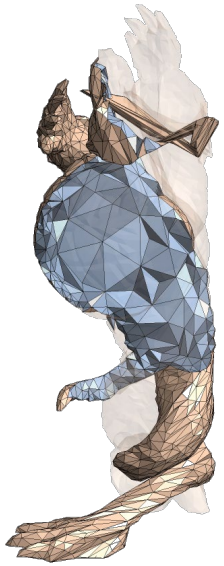
TLC

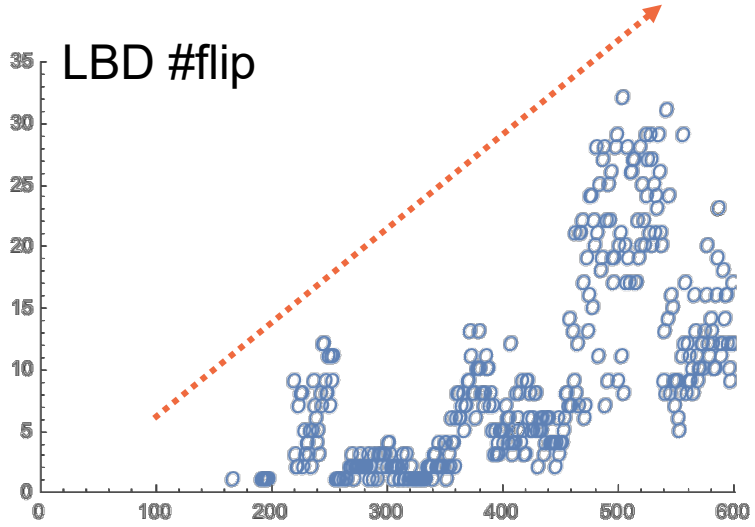
LBD [Kovalsky et al. 2015]

5

3D DEFORMATION

Deform





Tutte

804

TLC

LBD[Kovalsky et al. 2015]

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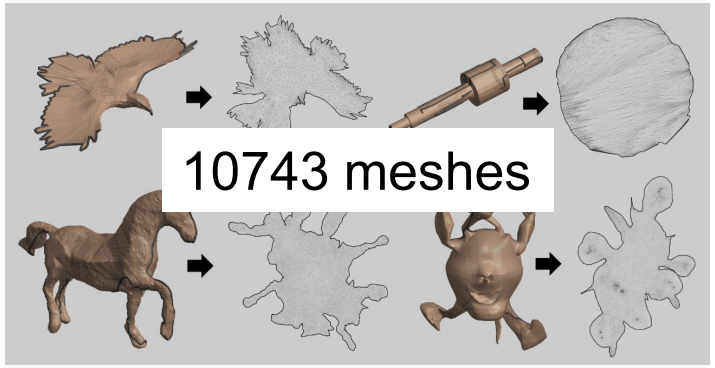
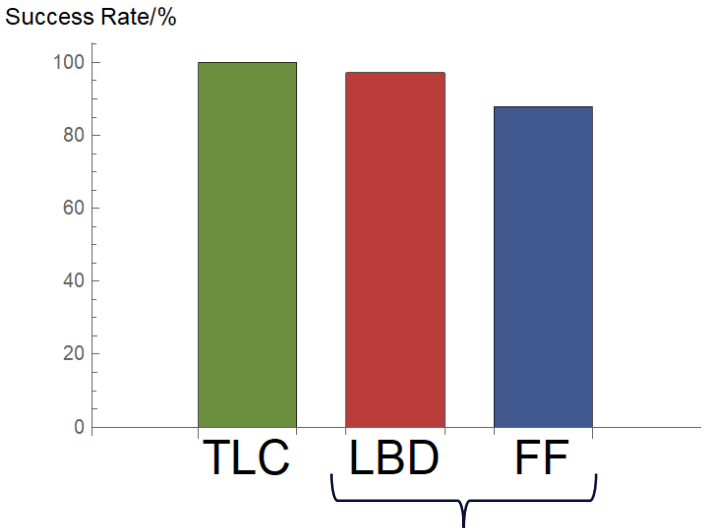
BENCHMARK SUMMARY

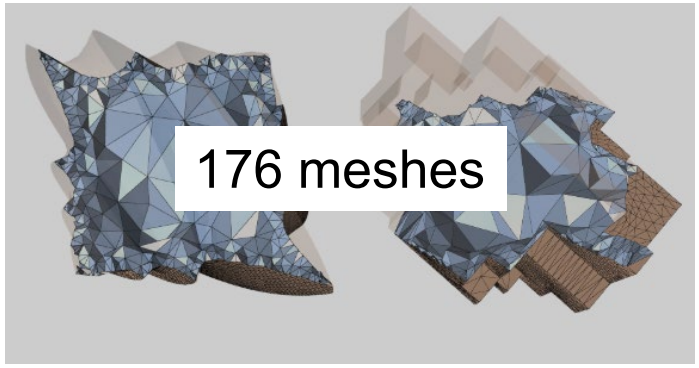
2D Parameterization

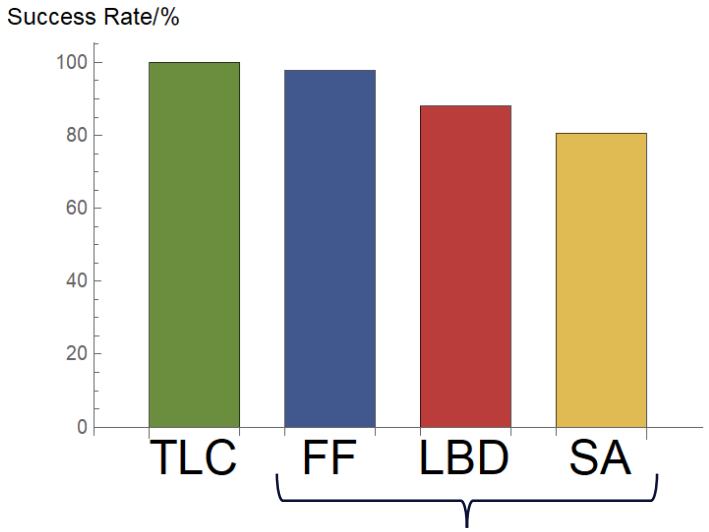




100% Mean: 93%

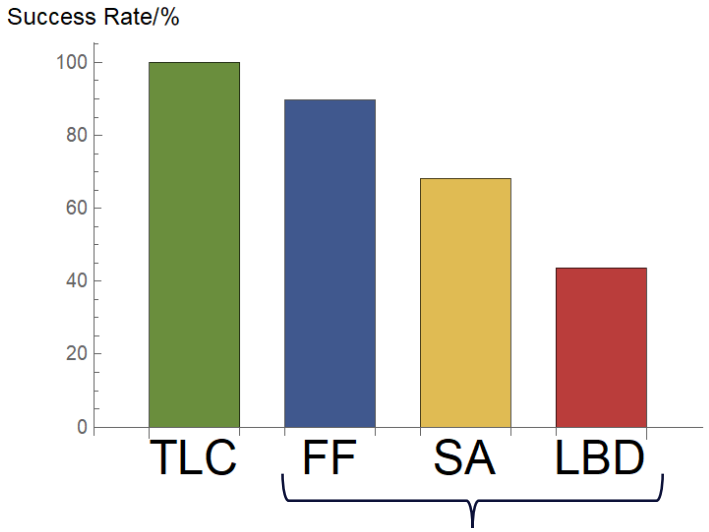
3D Parameterization





100% Mean: 89%

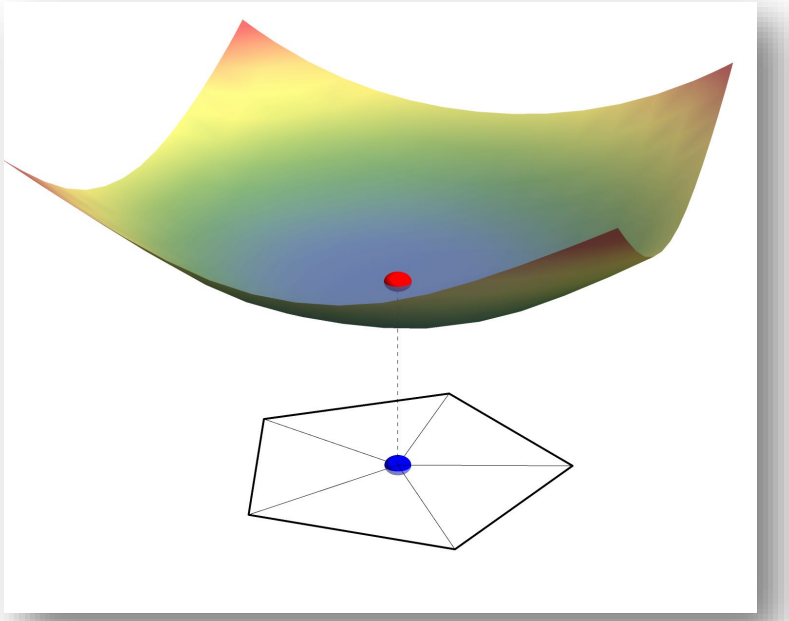
3D Deformation



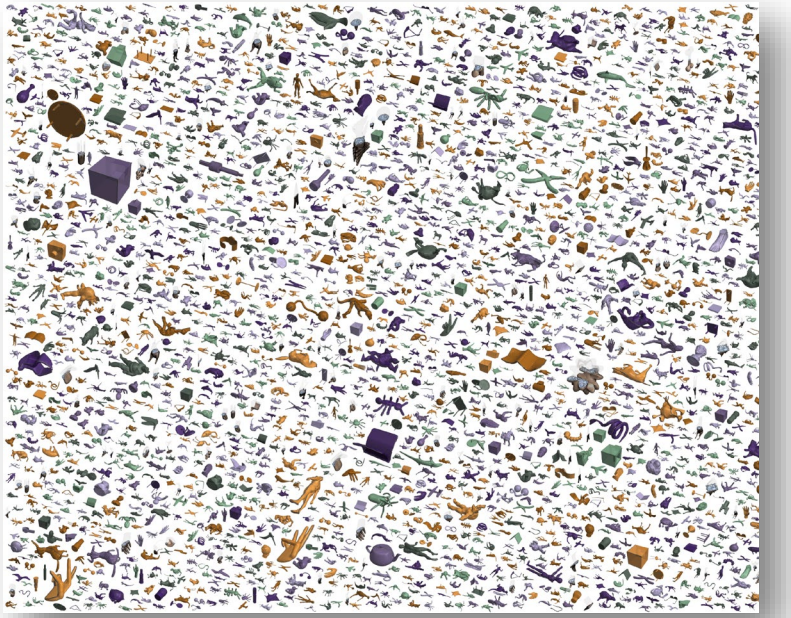
100% Mean: 67%

CONCLUSION

- New energy (TLC) for injectivity
  - guarantee injectivity at global minimum
  - high success rate in practice

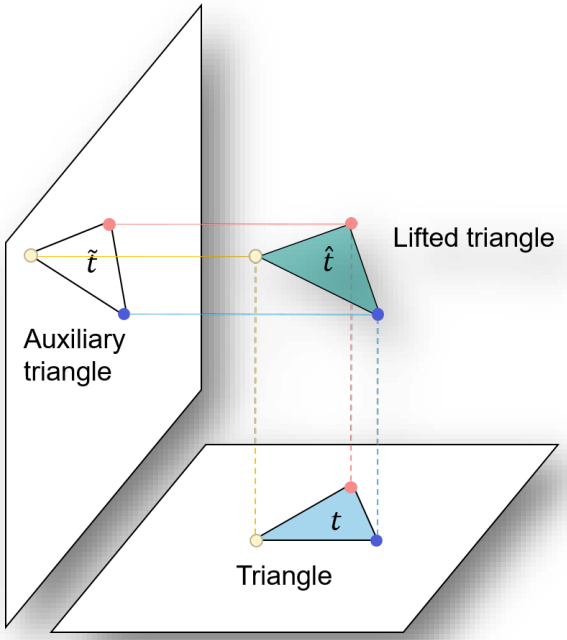


- Benchmark dataset for injective mappings
  - 10734 triangle meshes
  - 904 tetrahedron meshes



Future Directions

- injectivity at local minimum
- explore different types of auxiliary simplices



T L C

Code and Dataset

https://duxingyi-charles.github.io/publication/lifting-simplices-to-find-injectivity/