LIFTING SIMPLICES TO FIND INJECTIVITY

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Texture mapping

surface mesh

map

2D texture
MAPPING APPLICATIONS

Data transfer

source surface  Data  target domain  target surface

map
INJECTIVE (ONE-TO-ONE) MAPPING
INJECTIVE (ONE-TO-ONE) MAPPING
INJECTIVE (ONE-TO-ONE) MAPPING

injective

degeneracy
INJECTIVE (ONE-TO-ONE) MAPPING

injective

no flip/degeneracy

non-injective

degeneracy

flip
FIXED-BOUNDARY INJECTIVE MAPPINGS

**input:** (1) source mesh

**output:** mapping

- no flip/degeneracy

**(2) target boundary**
- correspond to source boundary
- no self-intersection
PREVIOUS WORK

• Minimize mapping distortion
  – [Schüller et al. 2013], [Liu et al. 2016], [Smith and Schaefer 2015], AMIPS
    [Fu et al. 2015], SLIM [Rabinovich et al. 2017], CM [Shtengel et al. 2017],
    [Claici et al. 2017], BCQN [Zhu et al. 2018], [Liu et al. 2018]
  – require injective mappings as initialization
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- Tutte Embedding [Tutte 1963]
  - guarantee injectivity for 2D convex domains
  - no guarantee for non-convex or 3D domains
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- Produce injective mappings
  - may change mesh structure [Agarwal et al. 2008], [Weber and Zorin 2014], [Campen et al. 2016], [Gu et al. 2018], [Shen et al. 2019]

```
injective
```

1024 vertices

1743 vertices
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• Produce injective mappings
  – may change mesh structure [Agarwal et al. 2008], [Weber and Zorin 2014], [Campen et al. 2016], [Gu et al. 2018], [Shen et al. 2019]
  – often fail on complex target domains [Aigerman and Lipman 2013], LBD [Kovalsky et al. 2015], SA [Fu and Liu 2016], FF [Su et al. 2019]
CONTRIBUTION

• New method to produce injective mappings
  – fixed-boundary domain in 2D/3D
  – maintain mesh structure

• New energy (Total Lifted Content, TLC)
  – theory: global minima are injective
  – practice: high success rate
CONTRIBUTION

• New method to produce injective mappings
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• New energy (Total Lifted Content, TLC)
  – theory: global minima are injective
  – practice: high success rate

• Benchmark dataset
  10734 triangle meshes
  904 tetrahedron meshes
TOTAL UNSIGNED AREA (TUA) [XU ET AL. 2011]

no flip

\[
\text{total area } (\quad) = \quad
\]

flip

\[
\text{total area } (\quad) = \quad + \quad
\]
PROBLEM OF TUA [XU ET AL. 2011]

- global minimum $\Rightarrow$ no flip

- non-injective global minimum

$$\text{total area ( } \quad \text{) } =$$
Auxiliary triangle
- fixed
- not degenerate

Lifted triangle
not degenerate

Triangle
degenerate

--- not smooth

no degeneracy

Lifting

degeneracy

non-injective global minimum

\[ \text{total area} = \]

\( (x_i, y_i, u_i, v_i) \)

\( (u_i, v_i) \)
TOTAL LIFTED CONTENT (TLC)

Auxiliary triangle
- fixed
- not degenerate

\[ \text{TLC} = \sum_{t \in \text{Mesh}} \text{LiftedContent}(t) \]

\[ \text{LiftedContent}(t) = \text{Area}(\hat{t}) \]
triangle

\[ \text{LiftedContent}(t) = \text{Volume}(\hat{t}) \]
tetrahedron

Total lifted content of a mesh

\[ \text{TLC}(\text{Mesh}) = \sum_{t \in \text{Mesh}} \text{LiftedContent}(t) \]

We use equilateral auxiliary simplices
TUA VS TLC

Total Unsigned Area

- not smooth
- non-injective global minimum

Total Lifted Content

- smooth
- global minimum is injective
PARAMETER $\alpha$

TLC with different $\alpha$

- $\alpha \rightarrow \infty$
  - minimize Dirichlet energy (auxiliary $\rightarrow$ target)
  - equilateral auxiliary simplices $\Rightarrow$ Tutte embedding

- $\alpha \rightarrow 0$
  - minimize Dirichlet energy (target $\rightarrow$ auxiliary)
  - 2D: MIPS energy

[Tutte, 2000]
BENCHMARK

[Aigerman and Lipman 2013]
[Weber and Zorin 2014]
[Fu et al. 2016]
Scaffold [Jiang et al. 2017]
[Liu et al. 2018]
FF [Su et al. 2019]
IPC [Li et al. 2020]
2D SIMPLE

Tutte TLC FF
TUA LBD
SA

flipped triangle

8 19 36 49

Tutte
TUA ($\alpha = 0$)
TLC ($\alpha = 10^{-6}$)
FF [Su et al. 2019]
LBD [Kovalsky et al. 2015]
SA [Fu and Liu 2016]
2D COMPLEX

Tutte

TLC

FF [Su et al. 2019]
3D PARAMETERIZATION

Tutte 3123

TLC

flipped tetrahedron

SA [Fu and Liu 2016]
3D DEFORMATION

Deform

Tutte 283
TLC
LBD [Kovalsky et al. 2015] 5

LBD #flip
frame number
3D DEFORMATION

Deform

Tutte
804

TLC

LBD [Kovalsky et al. 2015]
23

LBD #flip

frame number
BENCHMARK SUMMARY

**2D Parameterization**
- 10743 meshes
- Mean: 93%

**3D Parameterization**
- 176 meshes
- Mean: 89%

**3D Deformation**
- 728 meshes
- Mean: 67%
CONCLUSION

- New energy (TLC) for injectivity
  - guarantee injectivity at global minimum
  - high success rate in practice

- Benchmark dataset for injective mappings
  - 10734 triangle meshes
  - 904 tetrahedron meshes

Future Directions
- injectivity at local minimum
- explore different types of auxiliary simplices
TLC

Code and Dataset
https://duxingyi-charles.github.io/publication/lifting-simplices-to-find-injectivity/