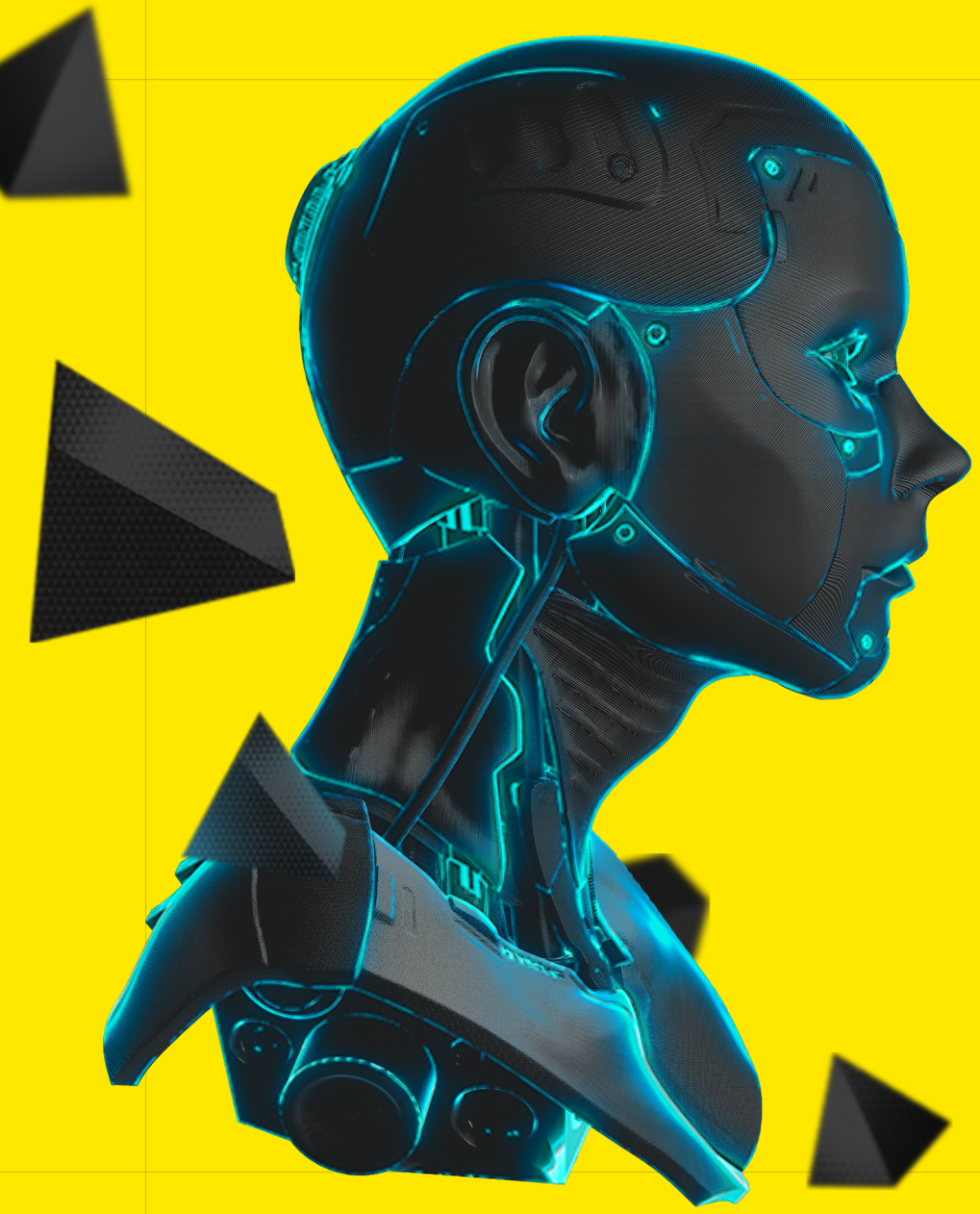


SIGGRAPH 2021

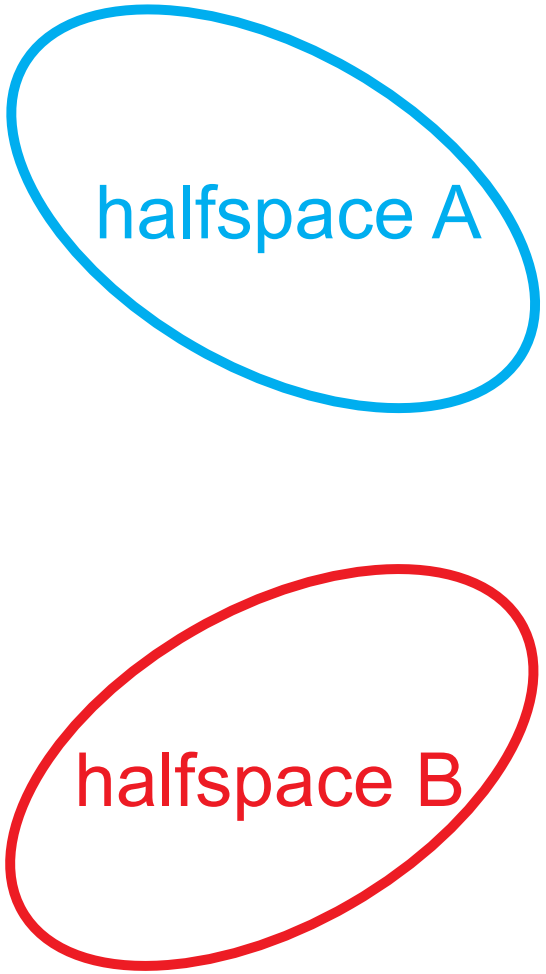
BOUNDARY-SAMPLED HALFSPACES:

A new representation for constructive solid modeling

Xingyi Du	Washington University in St. Louis
Qingnan Zhou	Adobe Research
Nathan Carr	Adobe Research
Tao Ju	Washington University in St. Louis

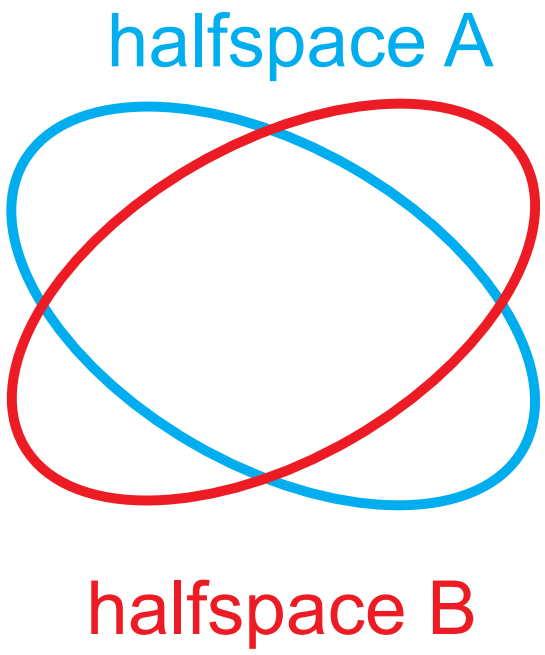
CONSTRUCTIVE SOLID GEOMETRY (CSG)

[Requicha and Voelcker 1977]



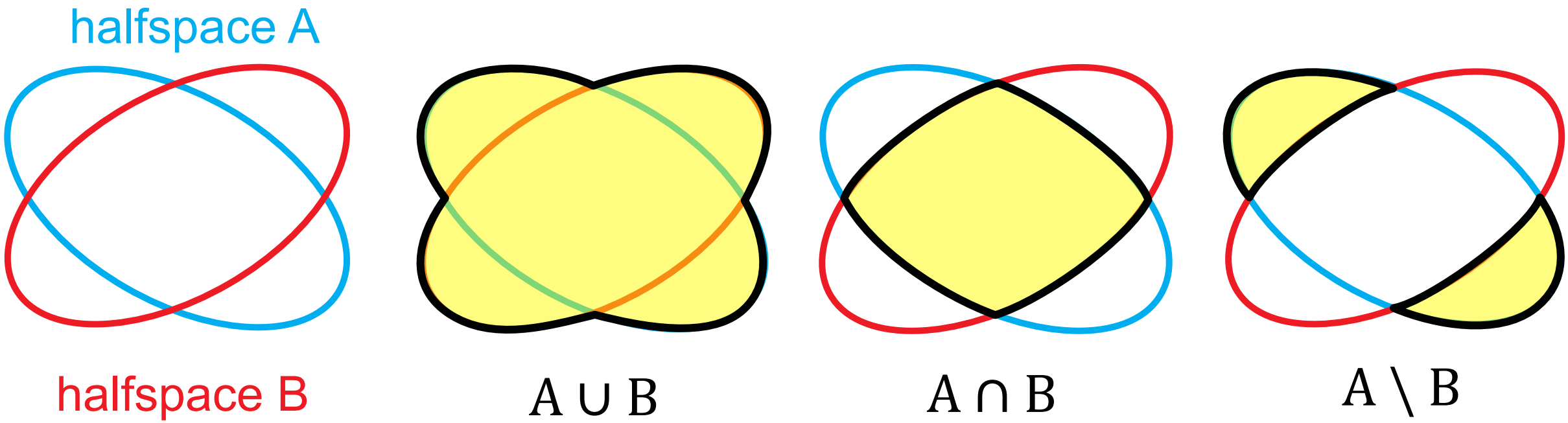
CONSTRUCTIVE SOLID GEOMETRY (CSG)

[Requicha and Voelcker 1977]

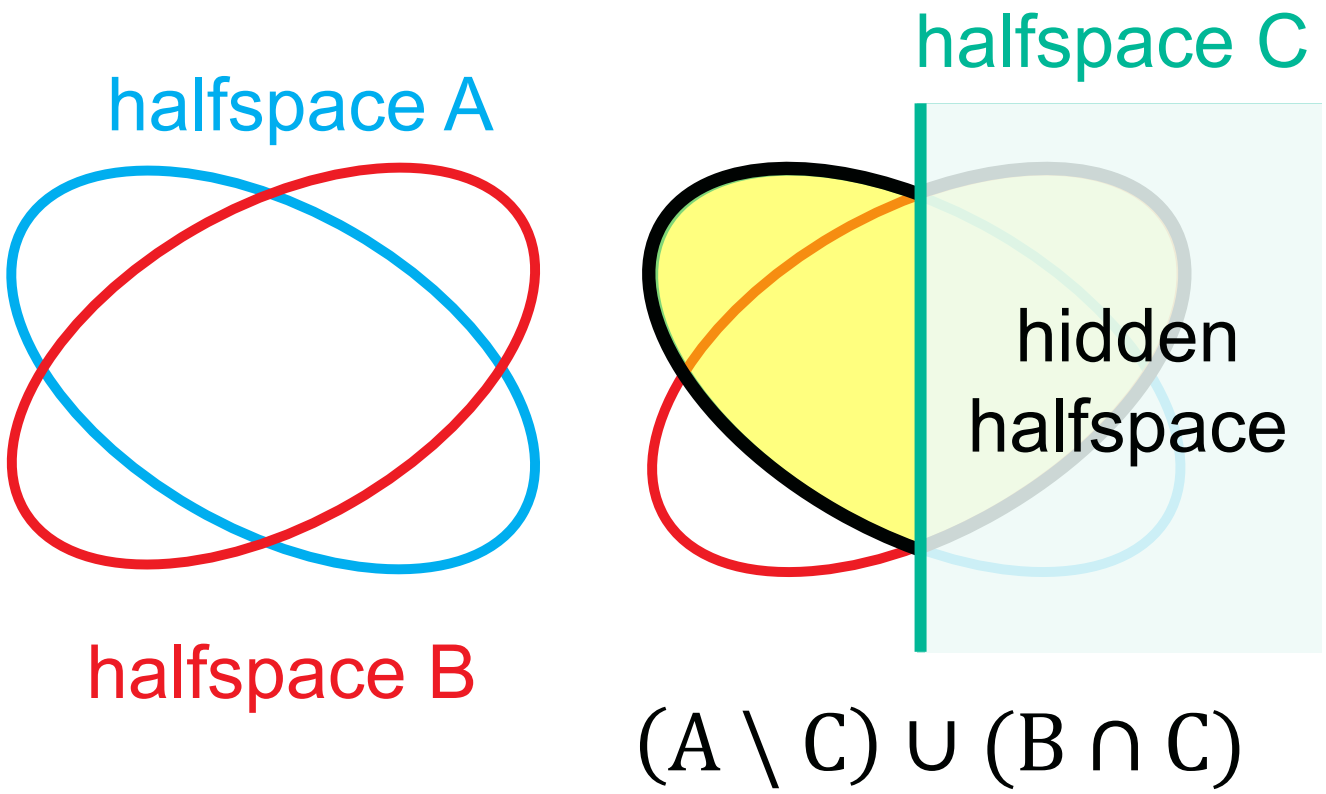


CONSTRUCTIVE SOLID GEOMETRY (CSG)

[Requicha and Voelcker 1977]

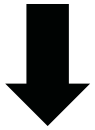


LIMITATIONS OF CSG



$$(A \setminus C) \cup (B \cap C)$$

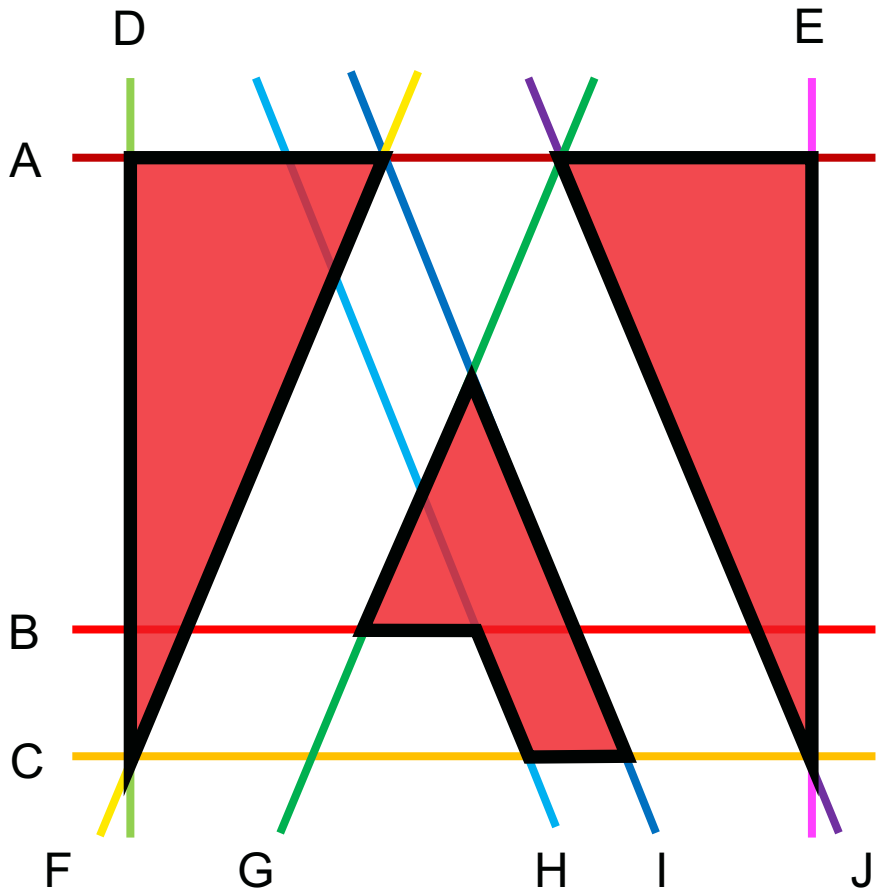
more complex boundary
halfspaces geometry



more complex
hidden halfspace geometry

[Shapiro and Vossler 1991]

LIMITATIONS OF CSG



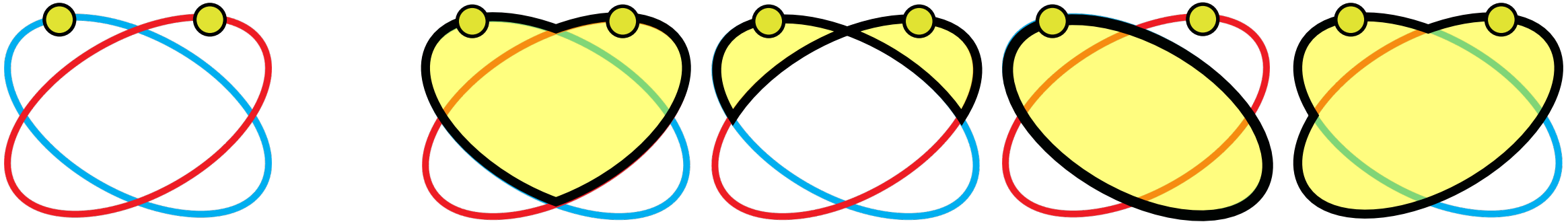
$$(A \cap D \cap F) \cup (A \cap E \cap J) \cup (B \cap G \cap I) \cup (C \cap H \cap I \cap G)$$

$$(A \cap D \cap E \cap C) \setminus ((A \cap \bar{F} \cap \bar{G} \cap C) \cup (A \cap \bar{I} \cap \bar{J} \cap C) \cup (G \cap \bar{B} \cap C \cap \bar{H}))$$

OUR CONTRIBUTION

- A new representation for solid models from halfspaces
  - No need for hidden halfspaces
  - Intuitive and light-weight
  - Easy for reverse engineering

BOUNDARY-SAMPLED HALFSPACES (BSH)



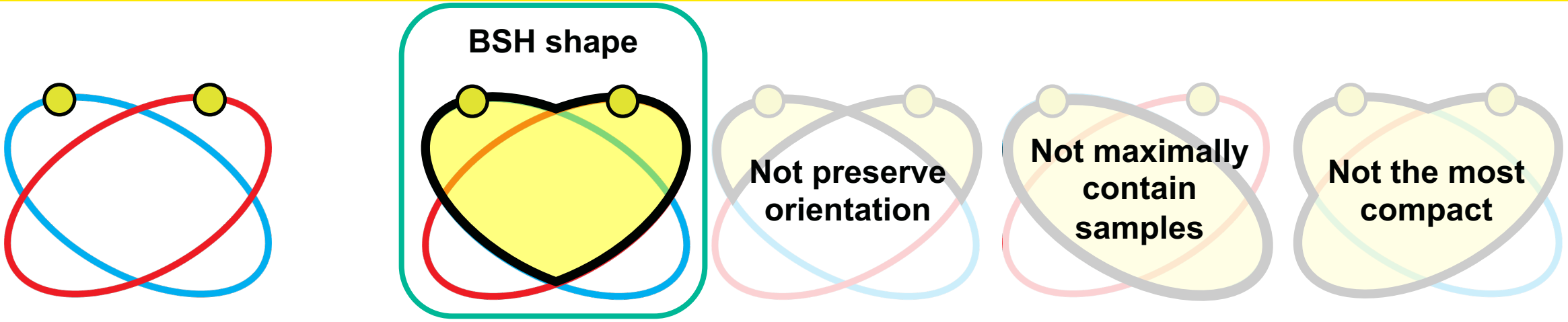
Representation:

- A set of halfspaces
- A set of samples

BSH shape:

- A subset of halfspace boundaries that bounds a solid:

BOUNDARY-SAMPLED HALFSPACES (BSH)



Representation:

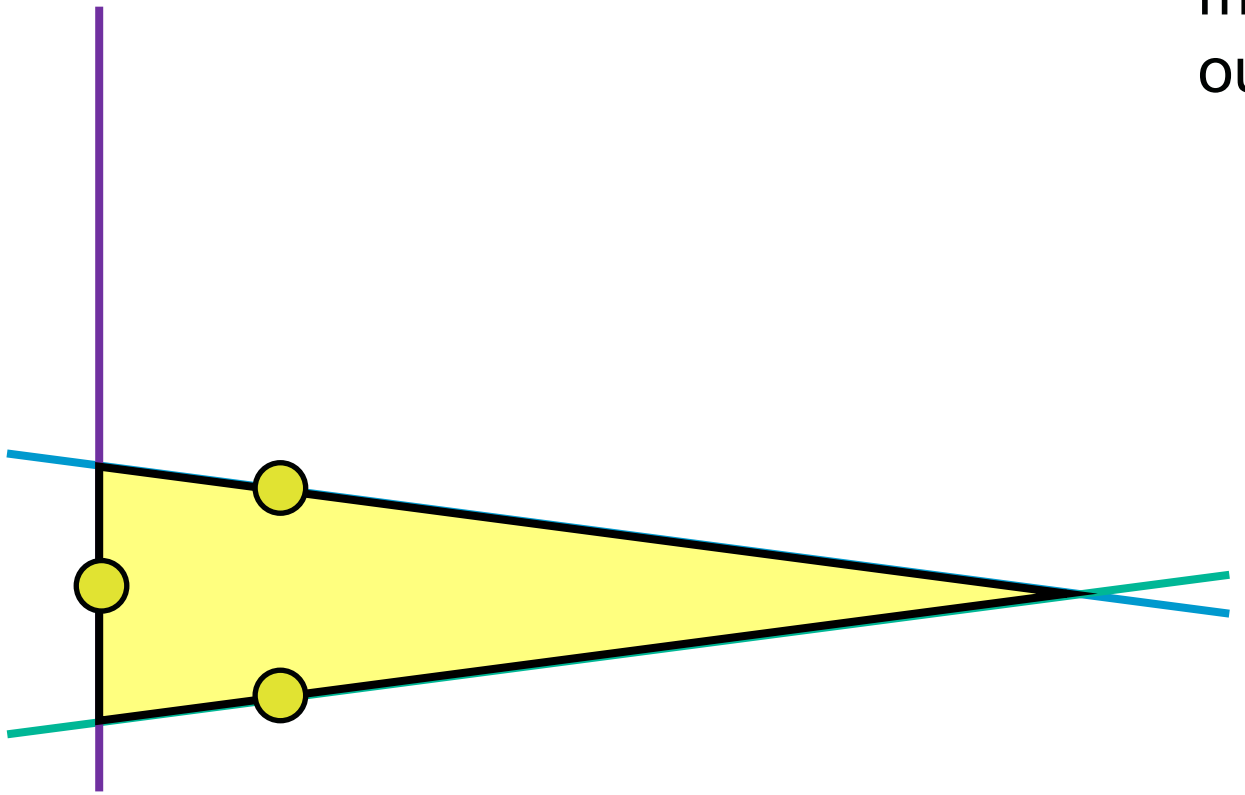
- A set of halfspaces
- A set of samples

BSH shape:

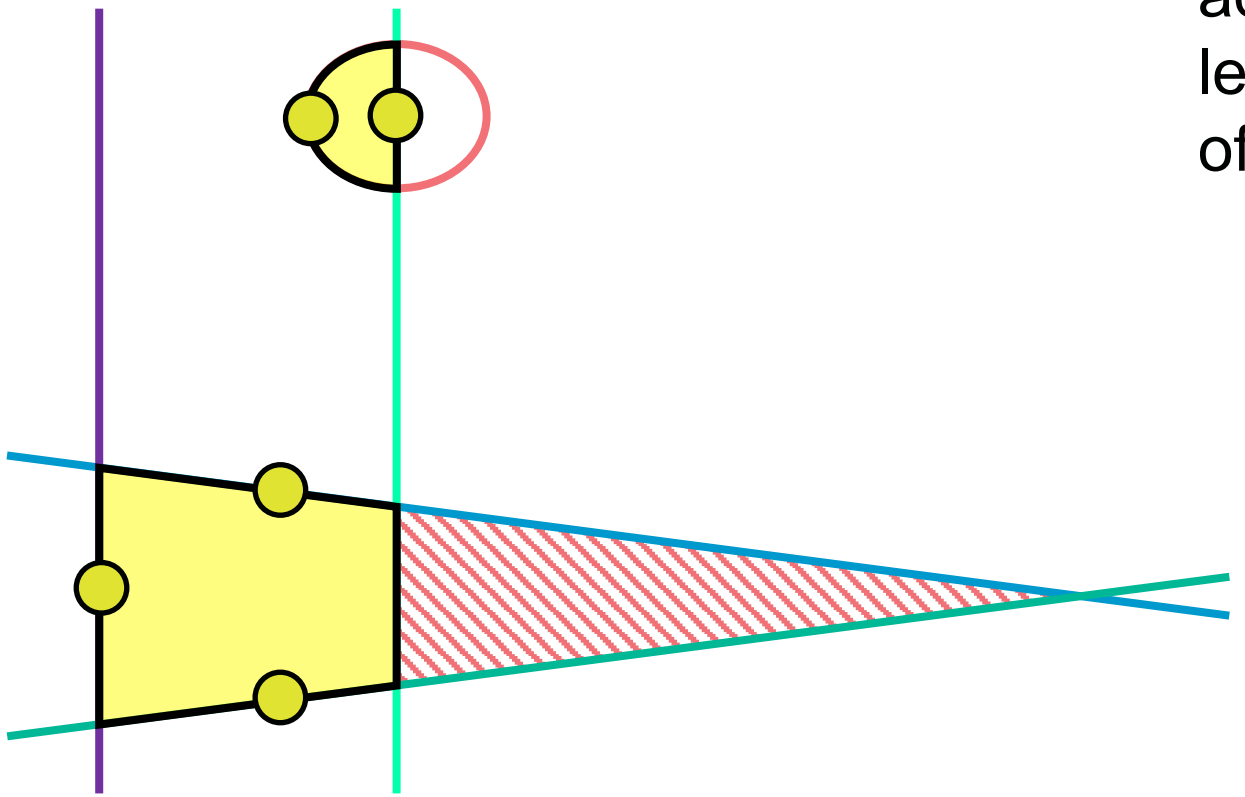
- A subset of halfspace boundaries that bounds a solid:
  - Preserves halfspace orientation
  - Contains as many samples as possible
  - Has the least boundary length (area)

CORNER CUTTING

model a triangle using
our representation

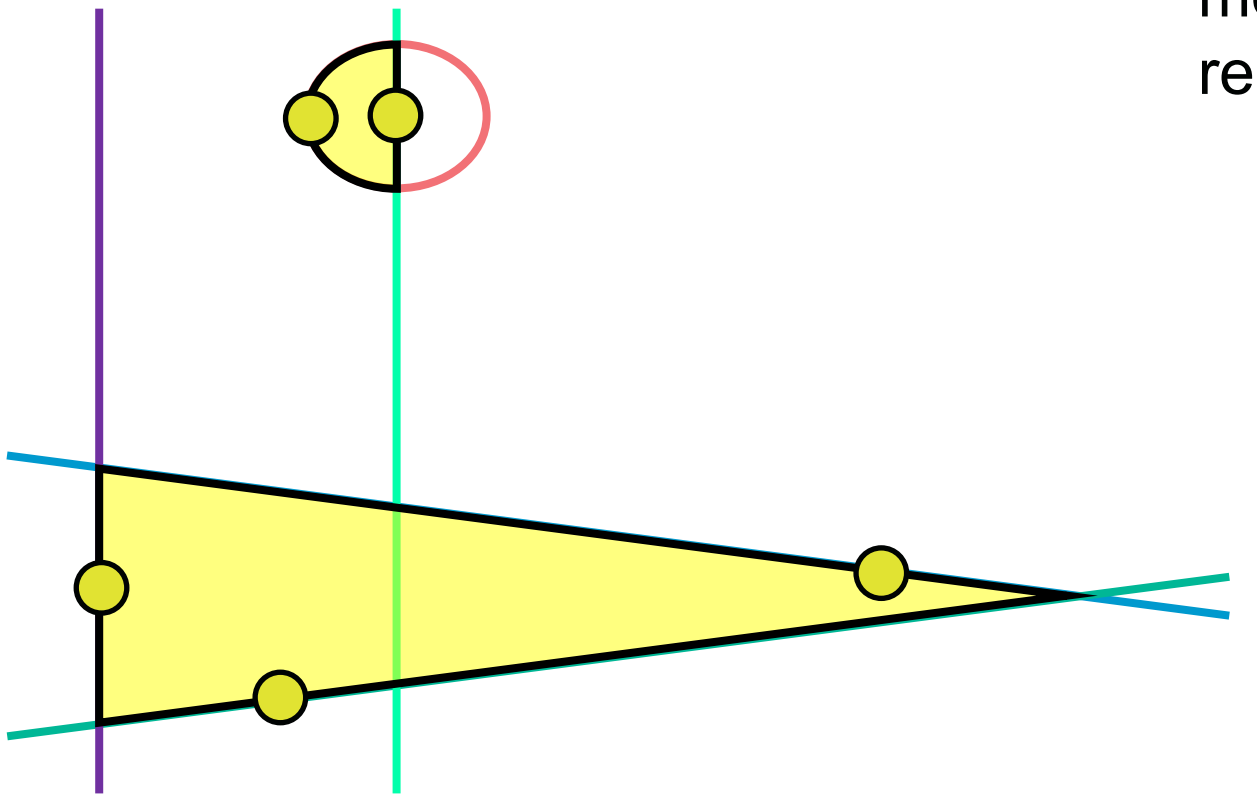


CORNER CUTTING



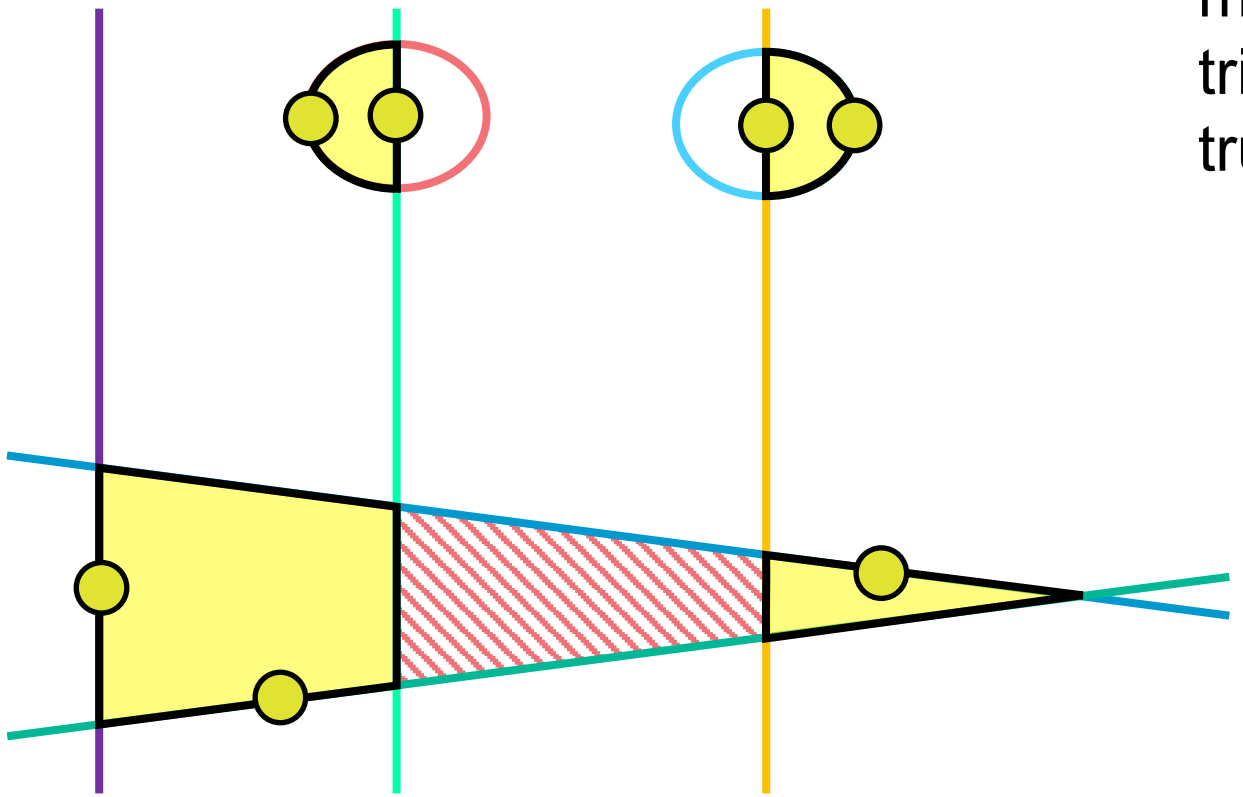
adding a semi-circle leads to corner cutting of the triangle

CORNER CUTTING



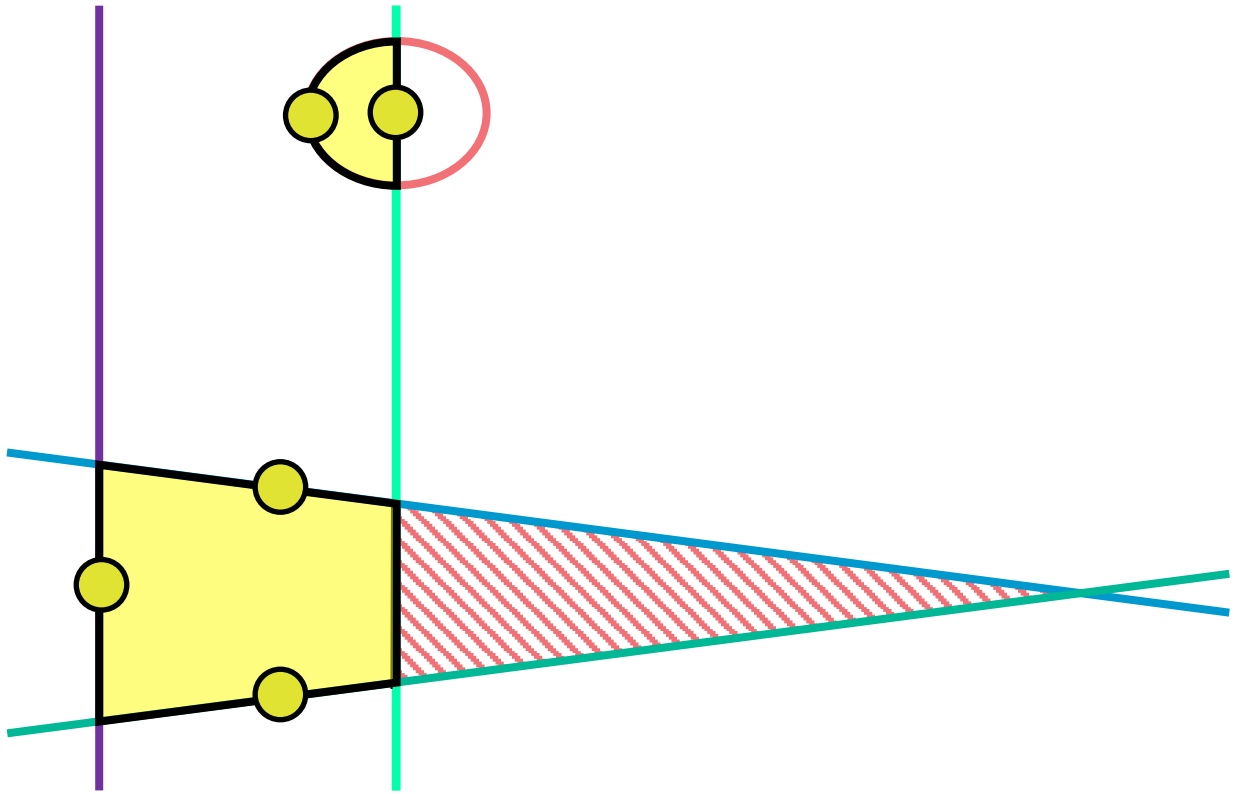
move a sample to
recover the corner

CORNER CUTTING

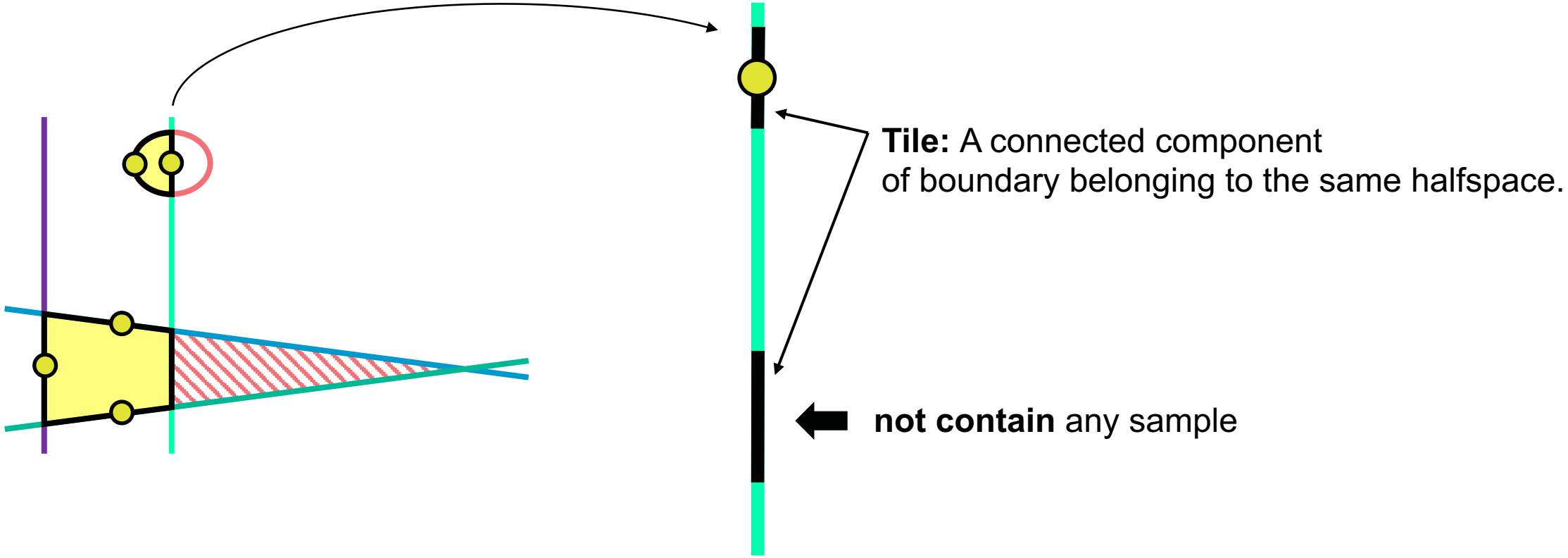


middle part of the triangle can also be truncated

SAMPLE CONNECTEDNESS



SAMPLE CONNECTEDNESS

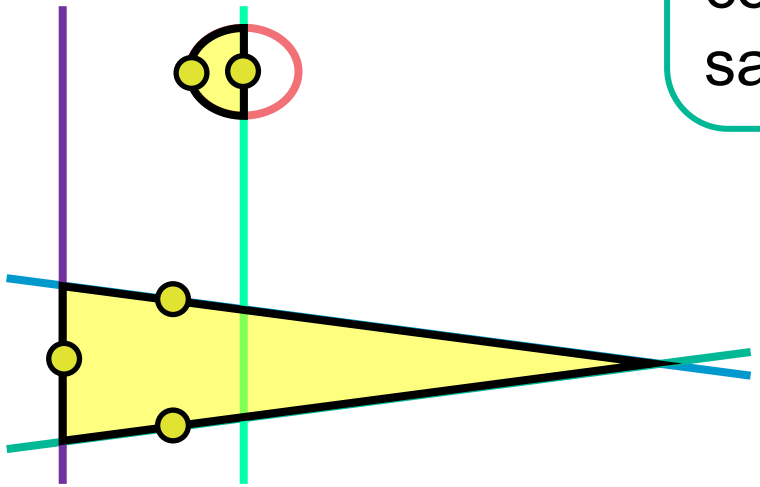


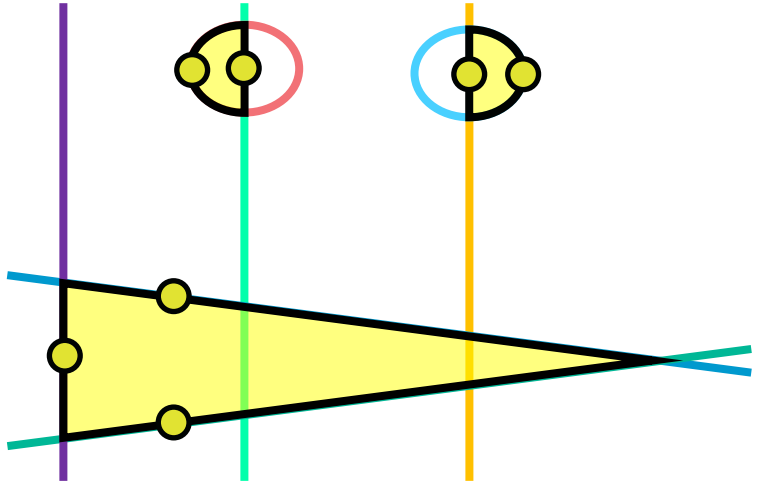
Tile: A connected component
of boundary belonging to the same halfspace.

not contain any sample

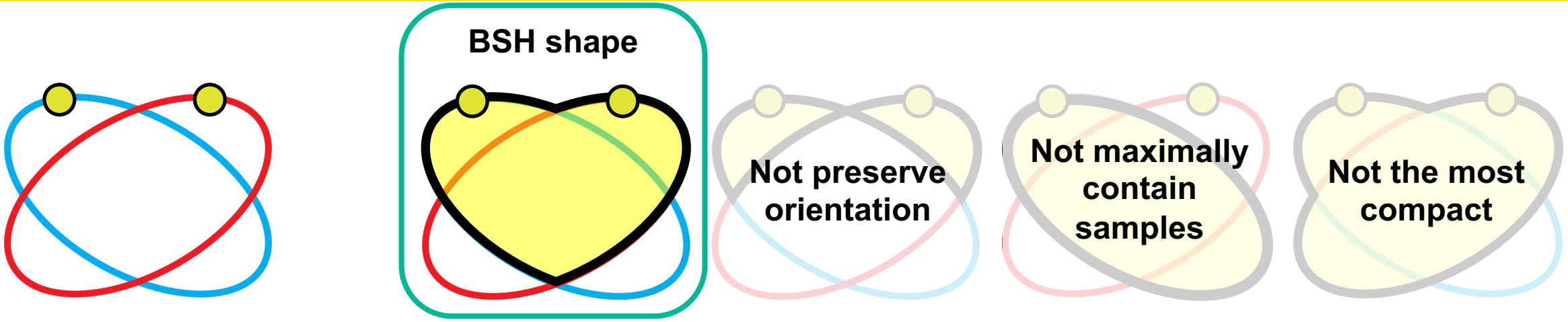
SAMPLE CONNECTEDNESS

sample connected: each boundary tile contains at least 1 sample





BOUNDARY-SAMPLED HALFSPACES (BSH)



Representation:

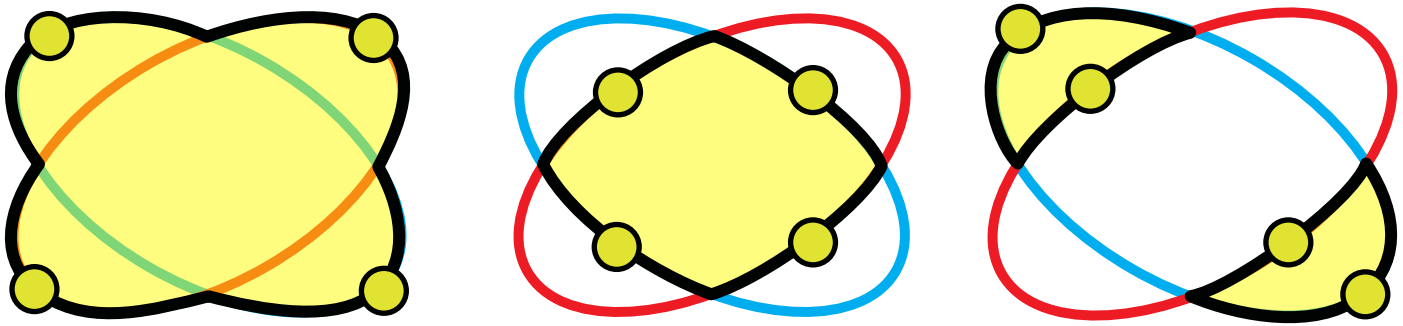
- A set of halfspaces
- A set of samples

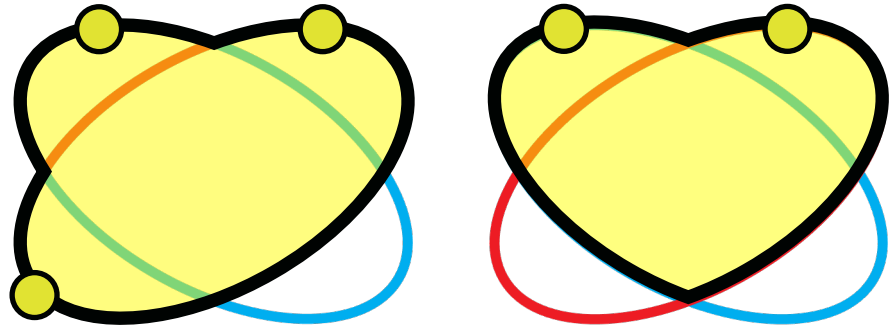
BSH shape:

- A subset of halfspace boundaries that bounds a solid:
  - Preserves halfspace orientation
  - Contains as many samples as possible
  - Has the least boundary length (area)
  - **Is sample connected**

DESCRIBABILITY

- BSH can describe any shape using only halfspaces that bound the shape and sufficient number of samples.



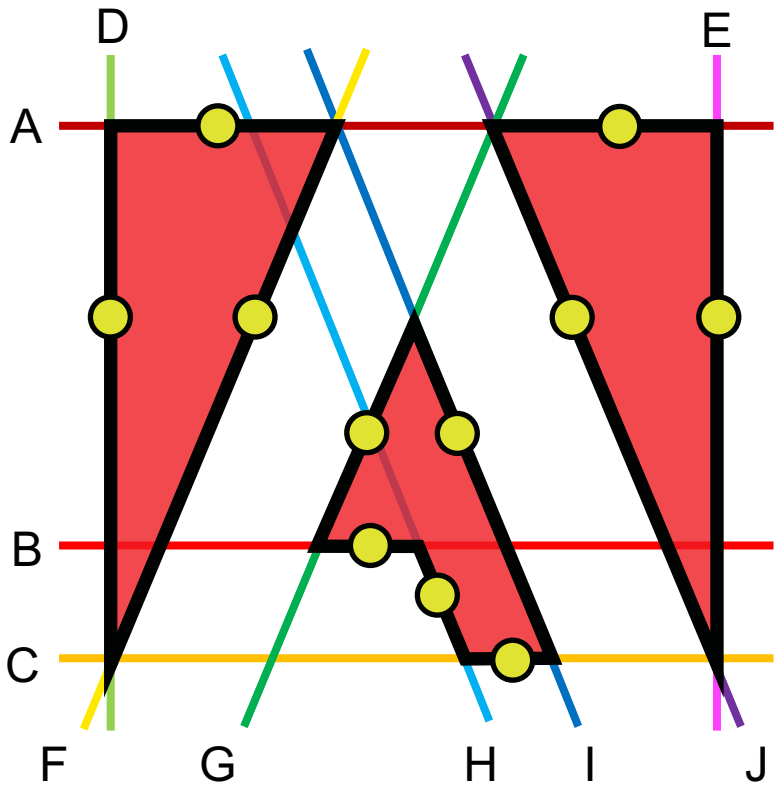


CSG needs hidden halfspaces

COMPARE BSH AND CSG

SIGGRAPH 2021

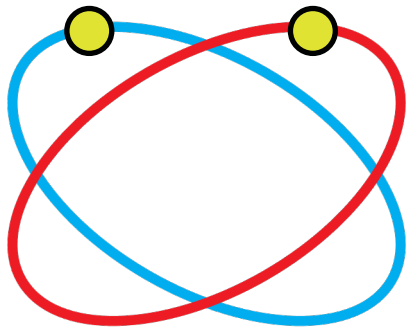
BSH



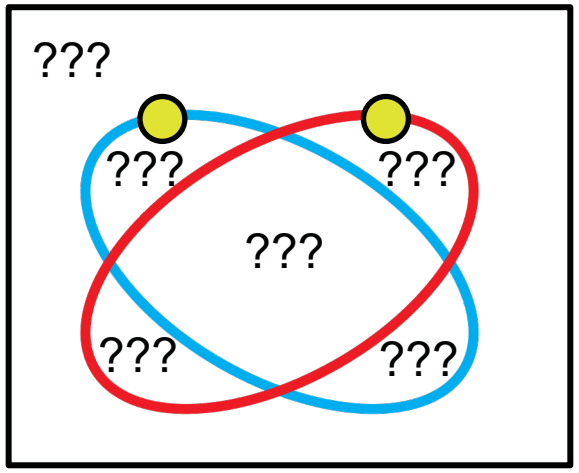
CSG

$$(A \cap D \cap F) \cup$$
$$(A \cap E \cap J) \cup$$
$$(B \cap G \cap I) \cup$$
$$(C \cap H \cap I \cap G)$$

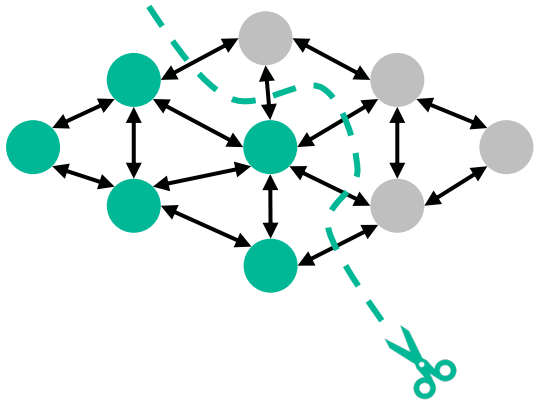
BOUNDARY EXTRACTION (W/O SAMPLE CONNECTEDNESS)



computing BSH
(w/o sample connectedness)



inside/outside labeling
of arrangement cells

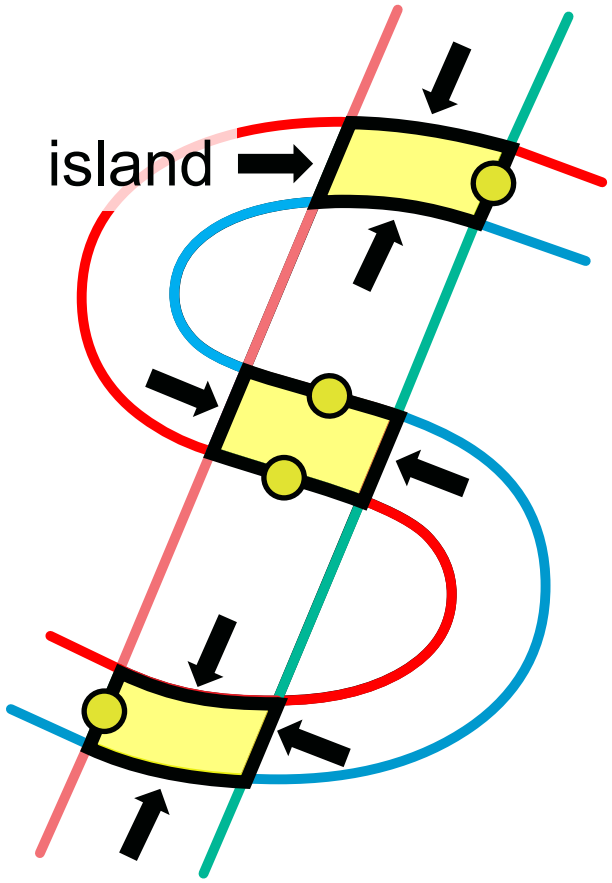


min-cut over
a weighted directed graph

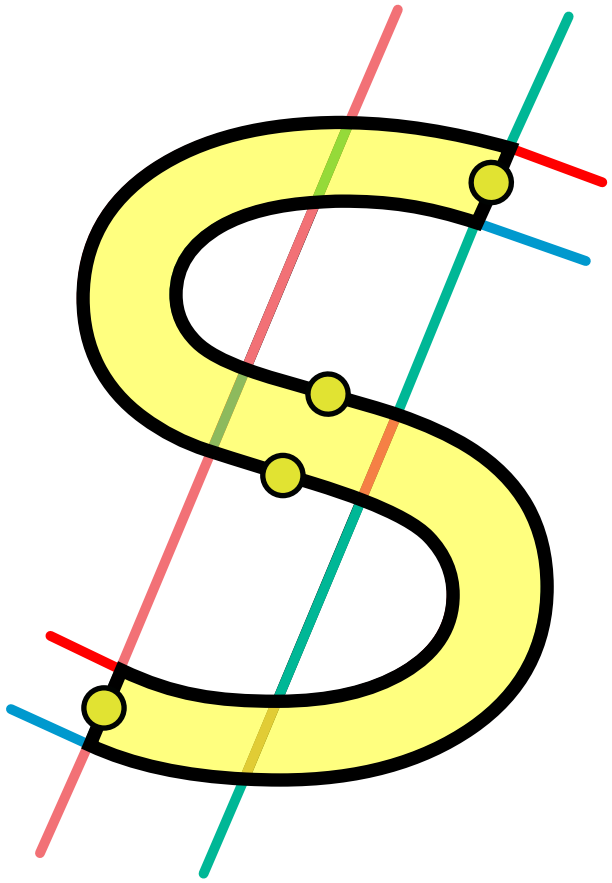
[Boulch et al. 2014] [Oesau et al. 2014]
[Chauve et al. 2010] [Verdie et al. 2015]
[Bauchet and Lafarge 2020]

BOUNDARY EXTRACTION (WITH SAMPLE CONNECTEDNESS)

- Graph-cut result may not be sample connected
  - Island: a tile that contains no sample



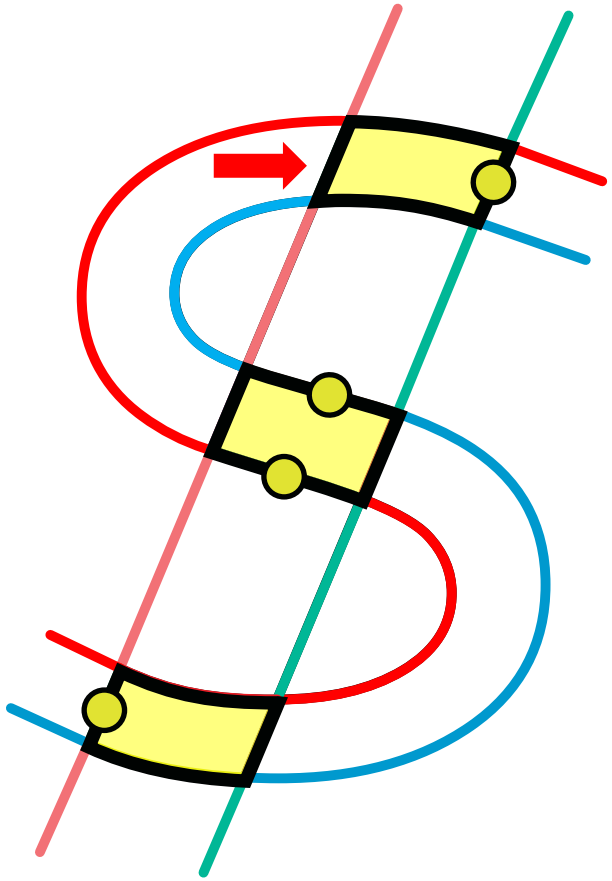
graph-cut result



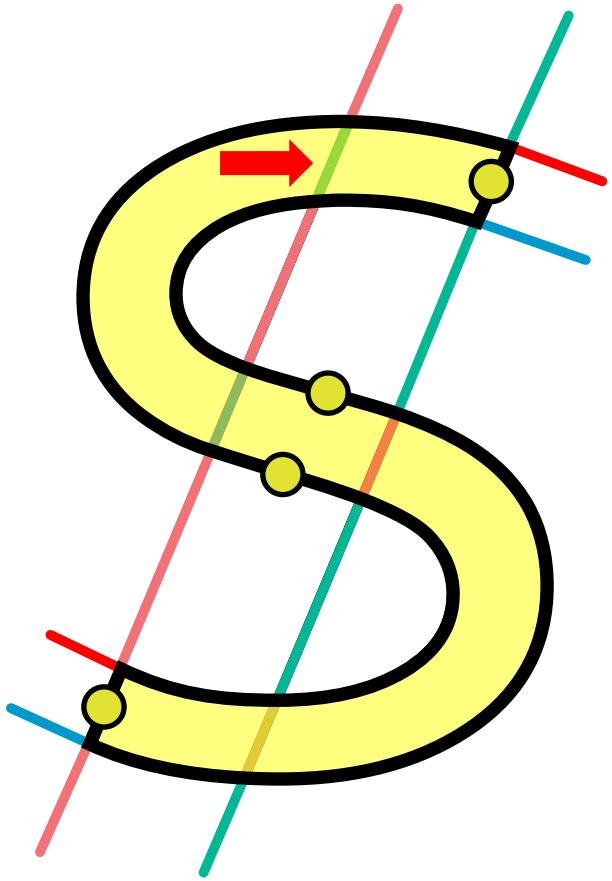
BSH

BOUNDARY EXTRACTION (WITH SAMPLE CONNECTEDNESS)

- Graph-cut result may not be sample connected
  - Island: a tile that contains no sample
- Observation: for each island, either the island or one of its adjacent patches on the arrangement does not lie on BSH.



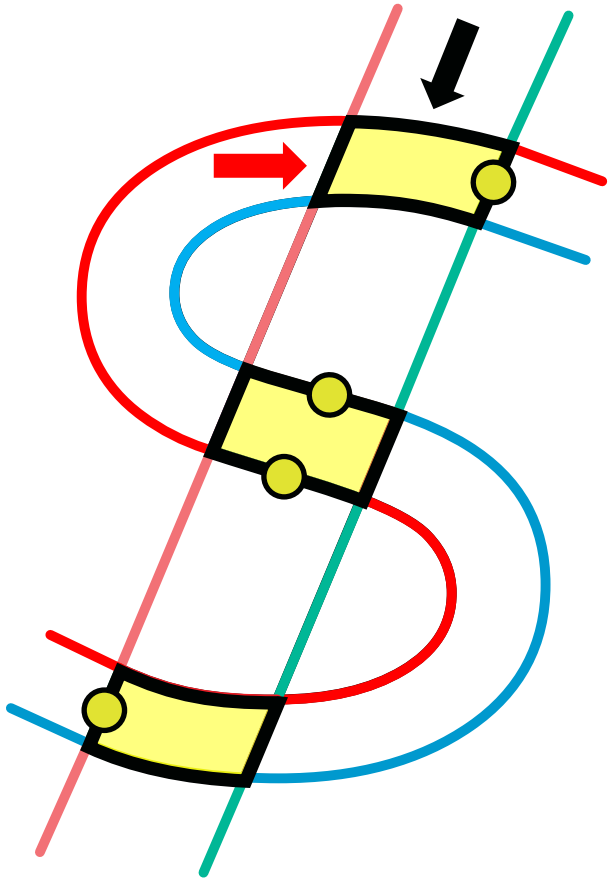
graph-cut result



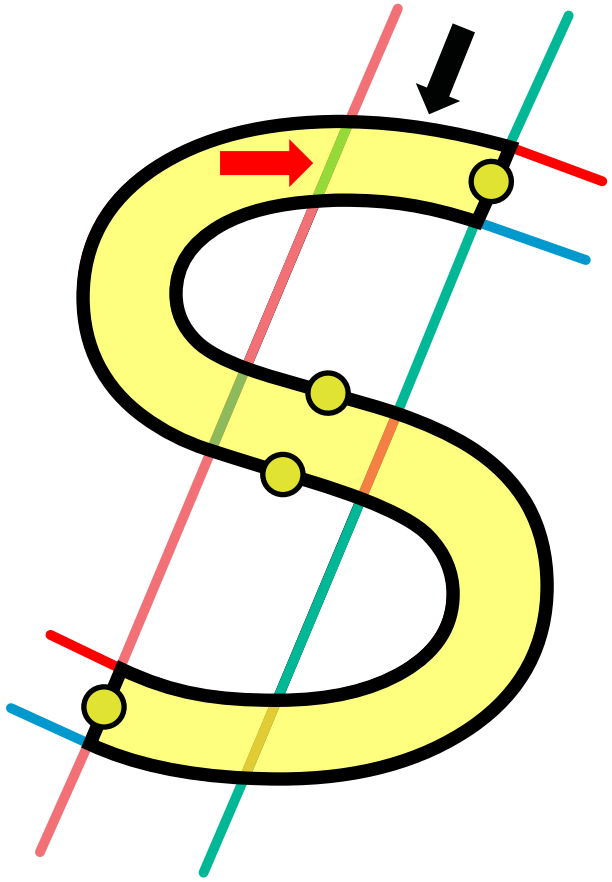
BSH

BOUNDARY EXTRACTION (WITH SAMPLE CONNECTEDNESS)

- Graph-cut result may not be sample connected
  - Island: a tile that contains no sample
- Observation: for each island, either the island or one of its adjacent patches on the arrangement does not lie on BSH.



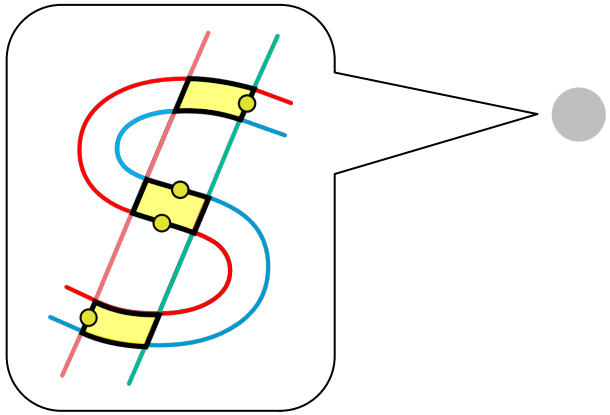
graph-cut result



BSH

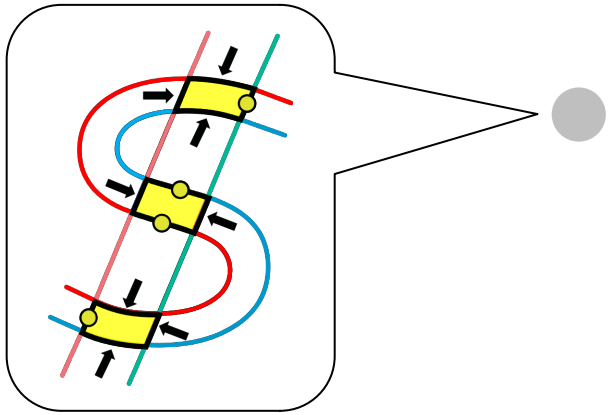
BOUNDARY EXTRACTION (WITH SAMPLE CONNECTEDNESS)

- A state-space search algorithm
  - Each “state” is a set of patches of arrangement that cannot be used (length set to infinity)



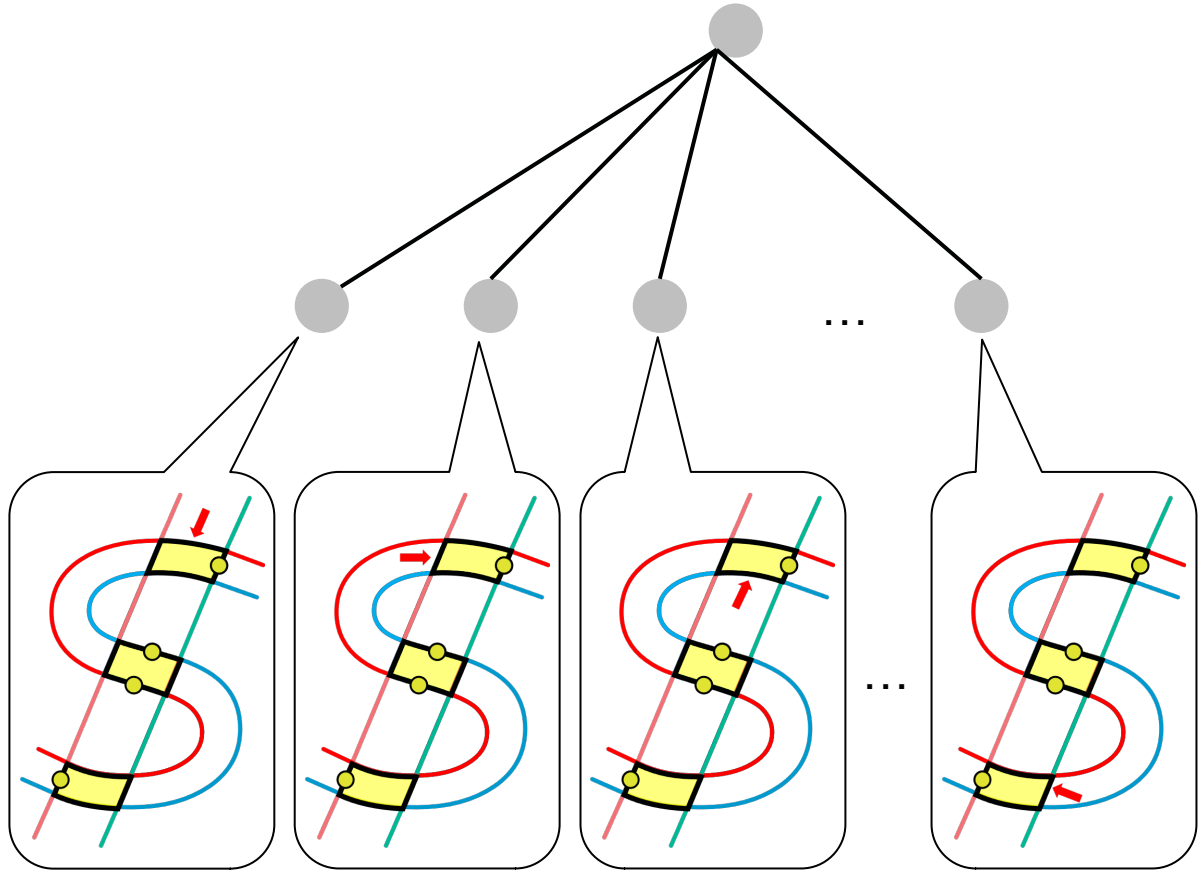
BOUNDARY EXTRACTION (WITH SAMPLE CONNECTEDNESS)

- A state-space search algorithm
  - Each “state” is a set of patches of arrangement that cannot be used (length set to infinity)
  - New states are expanded by computing graph-cut in the existing state and setting one of the islands or its adjacent patches to be unusable



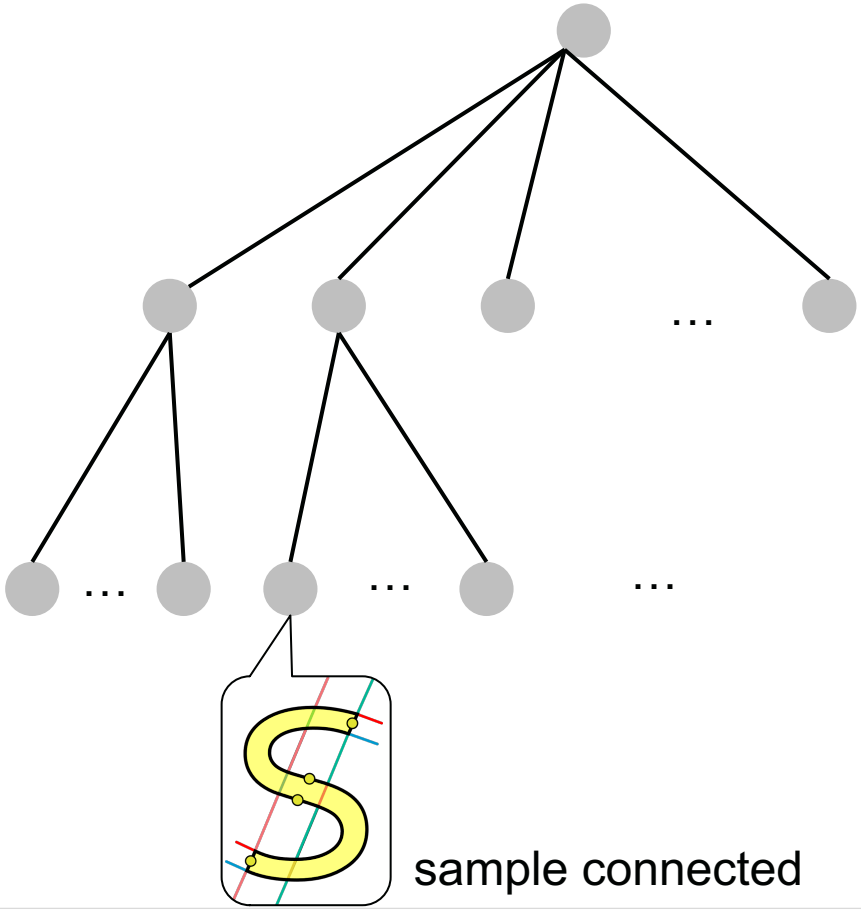
BOUNDARY EXTRACTION (WITH SAMPLE CONNECTEDNESS)

- A state-space search algorithm
  - Each “state” is a set of patches of arrangement that cannot be used (length set to infinity)
  - New states are expanded by computing graph-cut in the existing state and setting one of the islands or its adjacent patches to be unusable



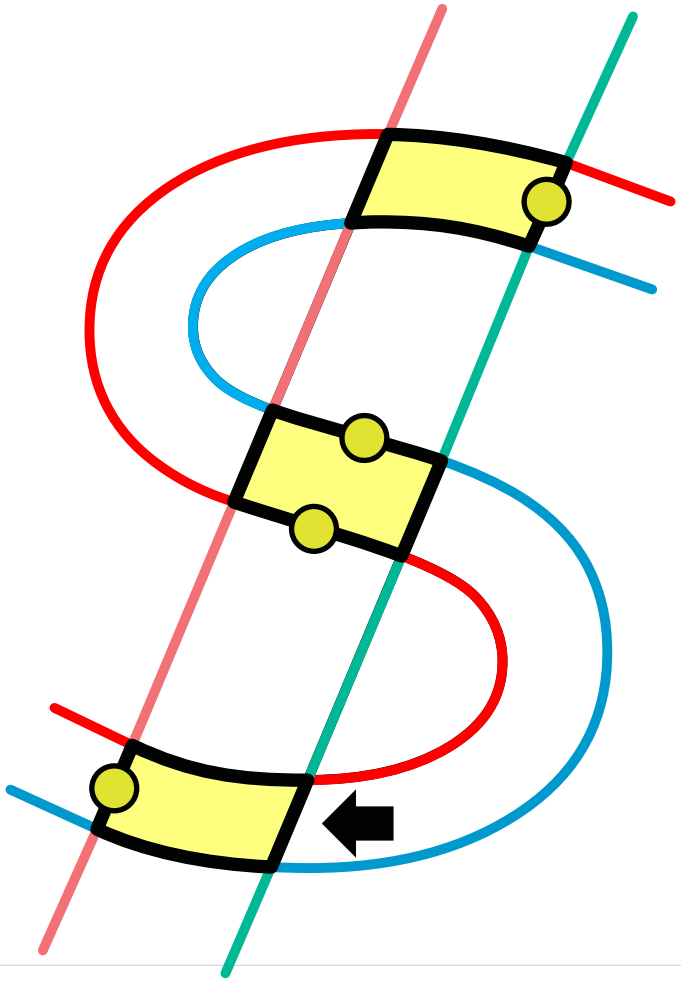
BOUNDARY EXTRACTION (WITH SAMPLE CONNECTEDNESS)

- A state-space search algorithm
  - Each “state” is a set of patches of arrangement that cannot be used (length set to infinity)
  - New states are expanded by computing graph-cut in the existing state and setting one of the islands or its adjacent patches to be unusable
  - States are explored in best-first order



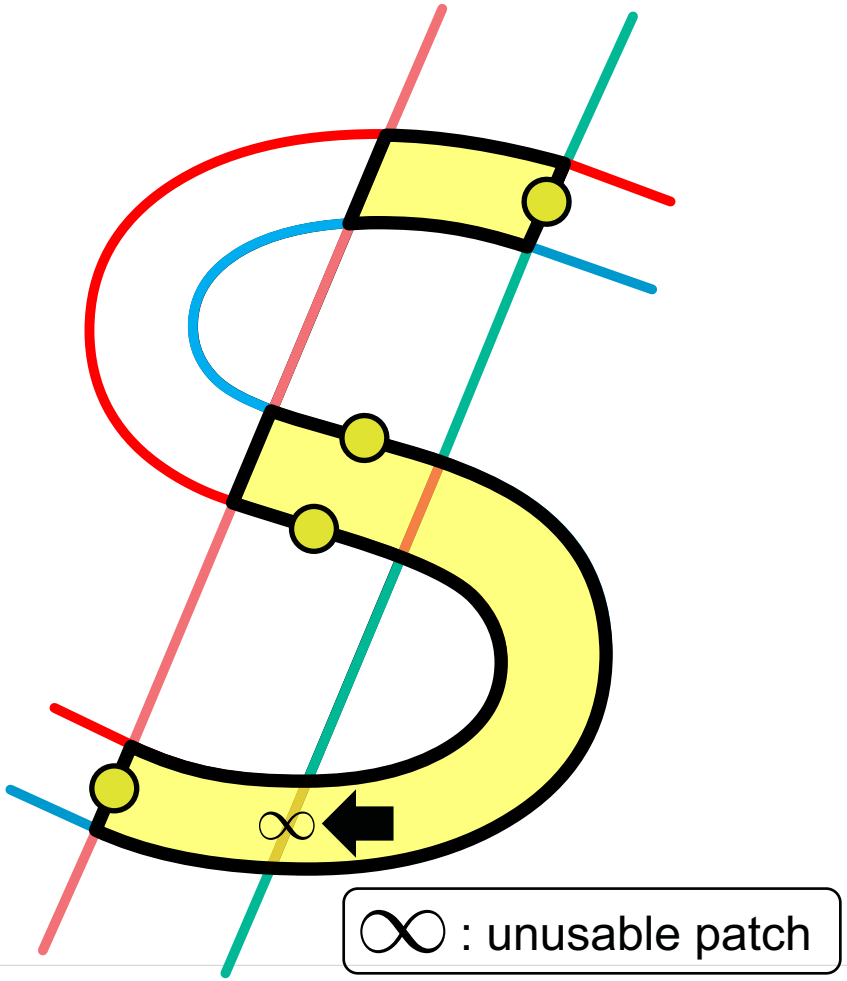
BOUNDARY EXTRACTION (WITH SAMPLE CONNECTEDNESS)

- A state-space search algorithm
  - Each “state” is a set of patches of arrangement that cannot be used (length set to infinity)
  - New states are expanded by computing graph-cut in the existing state and setting one of the islands or its adjacent patches to be unusable
  - States are explored in best-first order
- Greedy search algorithm
  - Only expand the best state



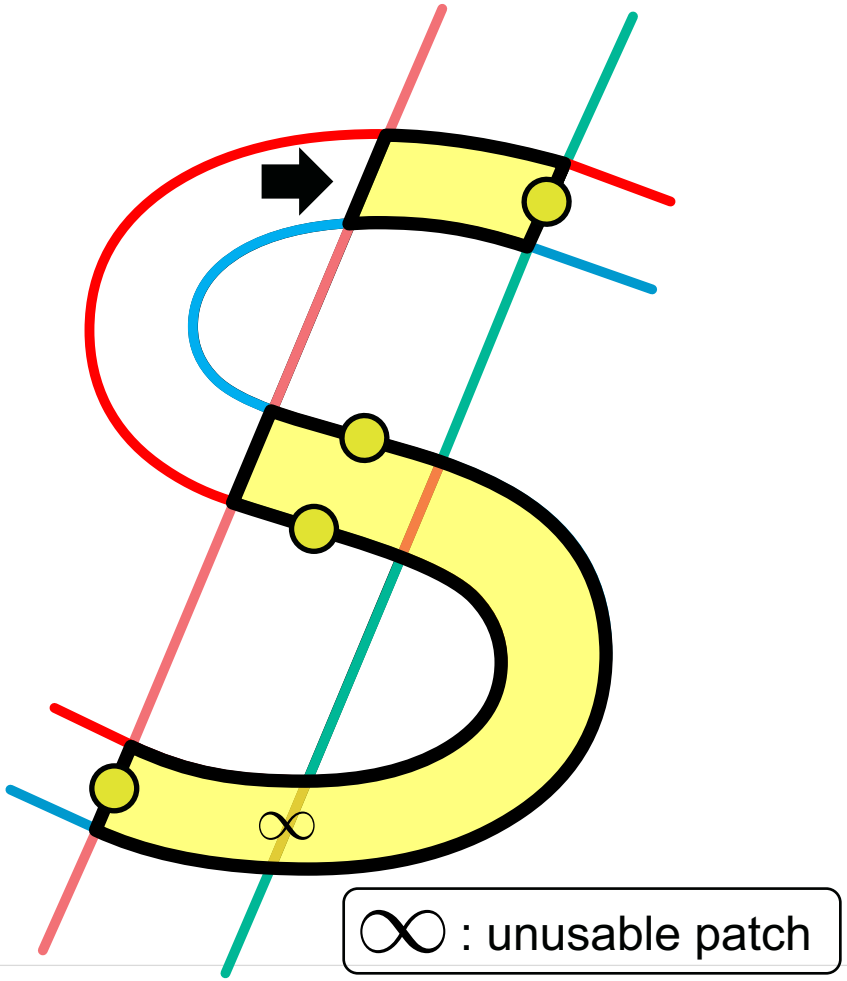
BOUNDARY EXTRACTION (WITH SAMPLE CONNECTEDNESS)

- A state-space search algorithm
  - Each “state” is a set of patches of arrangement that cannot be used (length set to infinity)
  - New states are expanded by computing graph-cut in the existing state and setting one of the islands or its adjacent patches to be unusable
  - States are explored in best-first order
- Greedy search algorithm
  - Only expand the best state



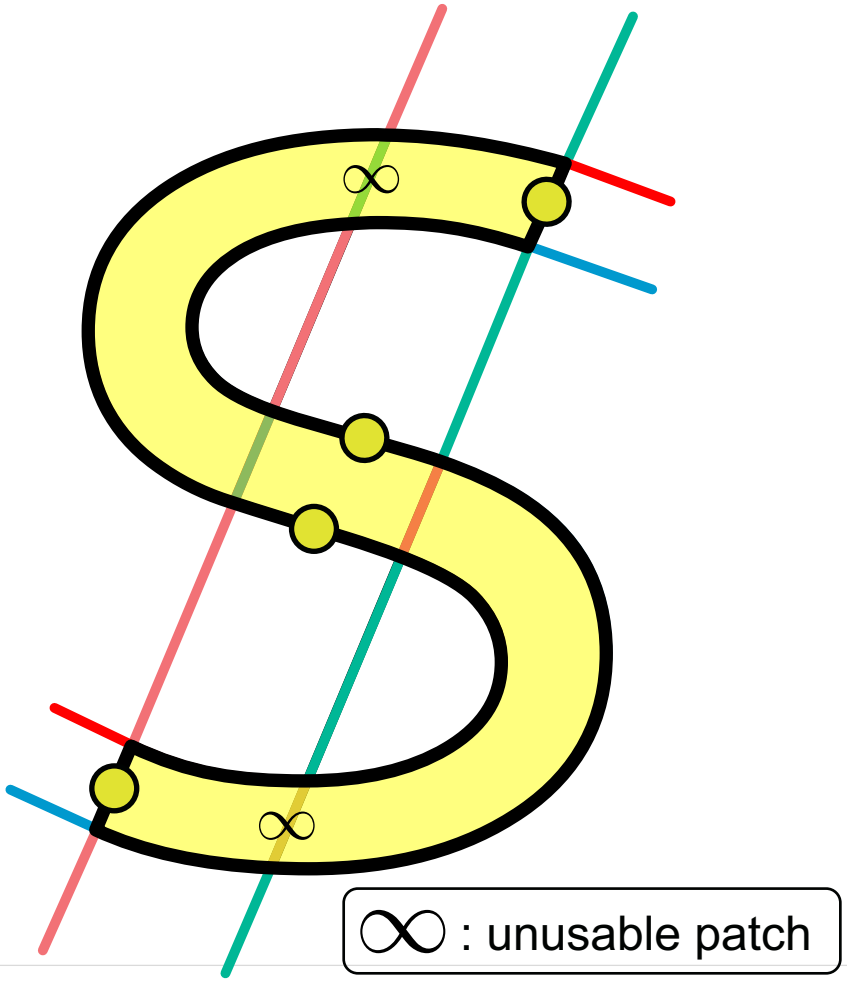
BOUNDARY EXTRACTION (WITH SAMPLE CONNECTEDNESS)

- A state-space search algorithm
  - Each “state” is a set of patches of arrangement that cannot be used (length set to infinity)
  - New states are expanded by computing graph-cut in the existing state and setting one of the islands or its adjacent patches to be unusable
  - States are explored in best-first order
- Greedy search algorithm
  - Only expand the best state



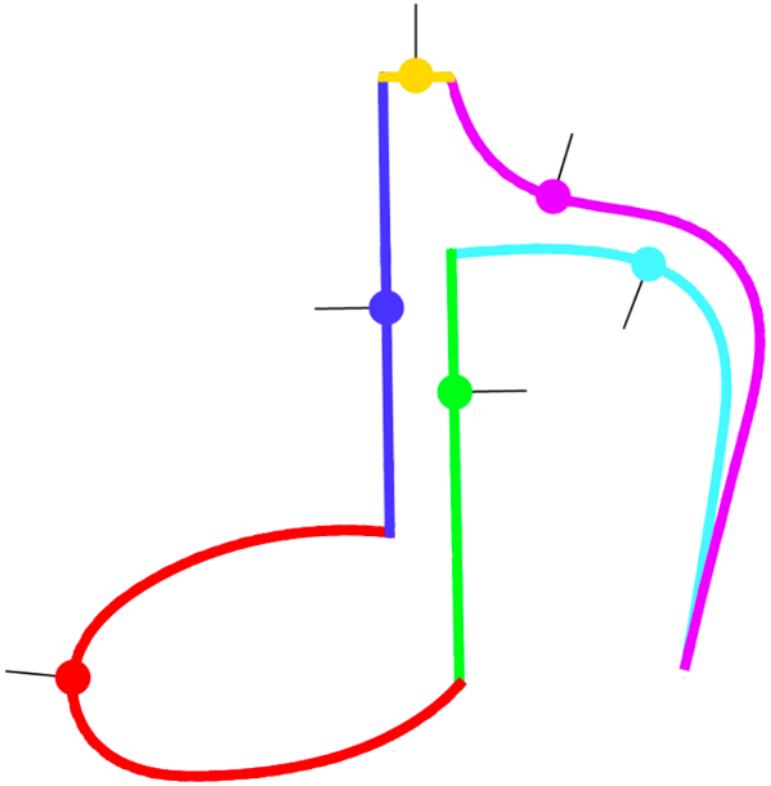
BOUNDARY EXTRACTION (WITH SAMPLE CONNECTEDNESS)

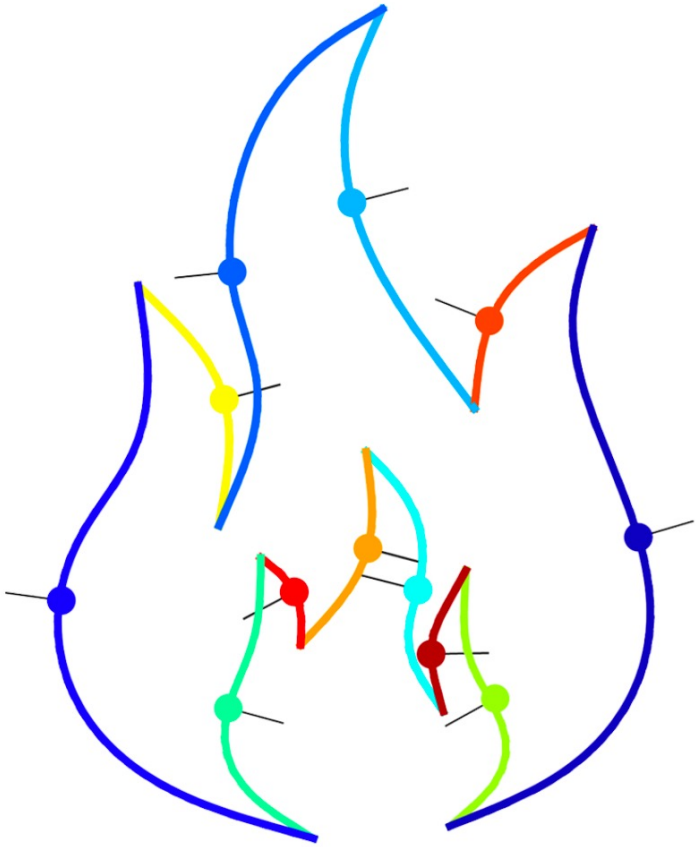
- A state-space search algorithm
  - Each “state” is a set of patches of arrangement that cannot be used (length set to infinity)
  - New states are expanded by computing graph-cut in the existing state and setting one of the islands or its adjacent patches to be unusable
  - States are explored in best-first order
- Greedy search algorithm
  - Only expand the best state

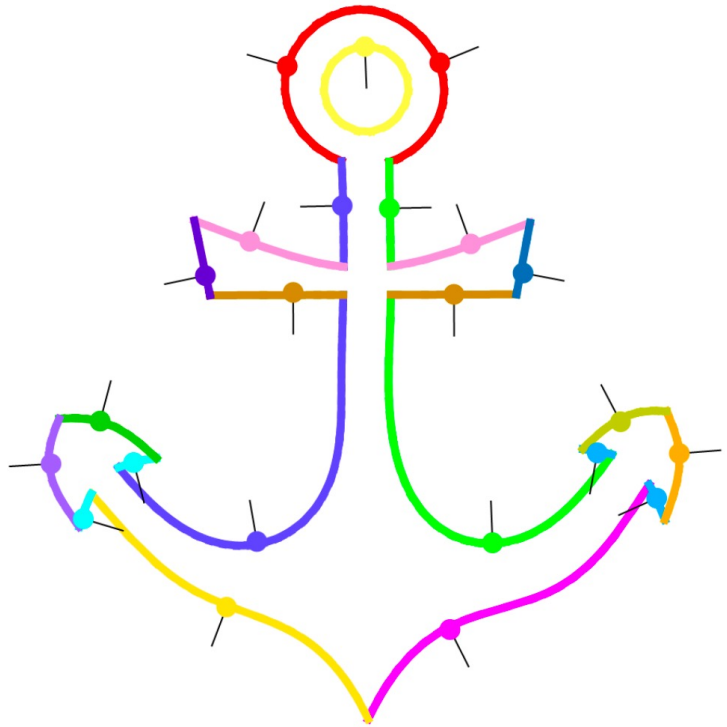


EXAMPLES

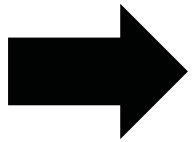
SIGGRAPH 2021





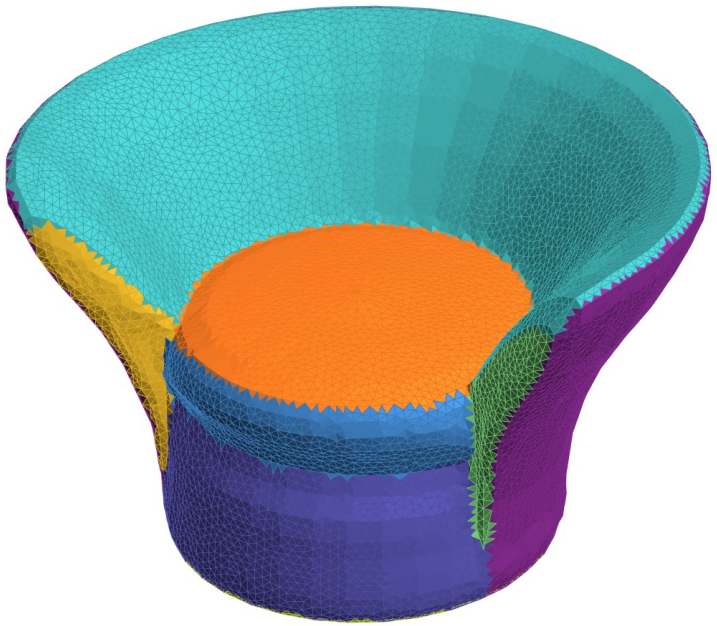


REVERSE ENGINEERING

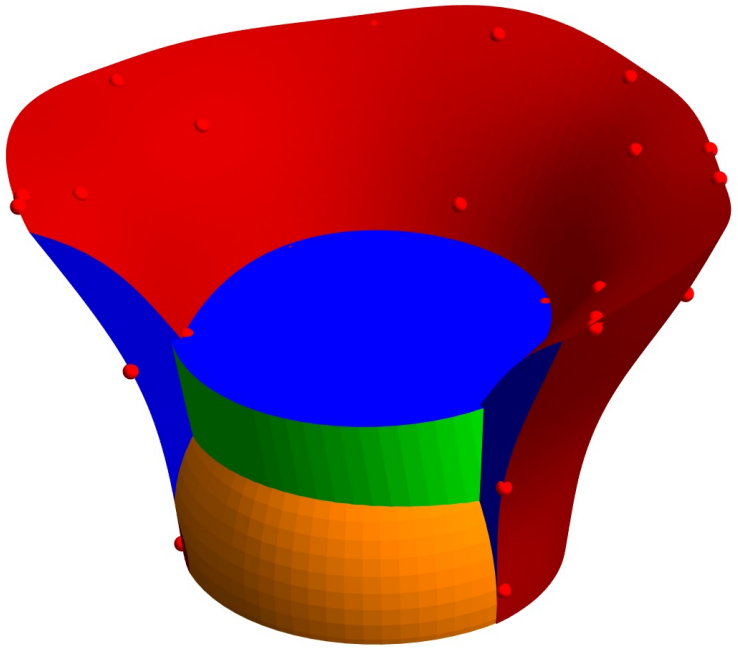
Meshes  BSH

- halfspaces
- samples

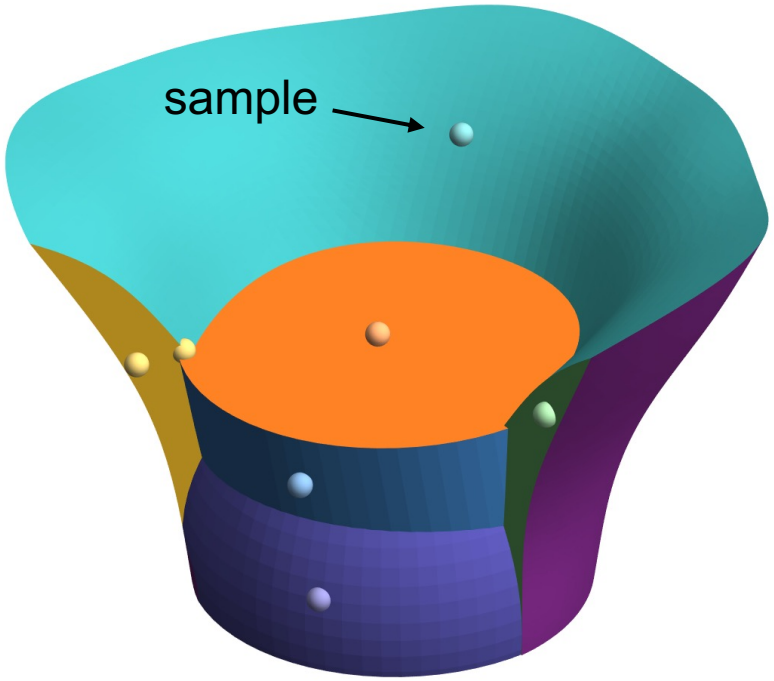
REVERSE ENGINEERING



Mesh segmentation



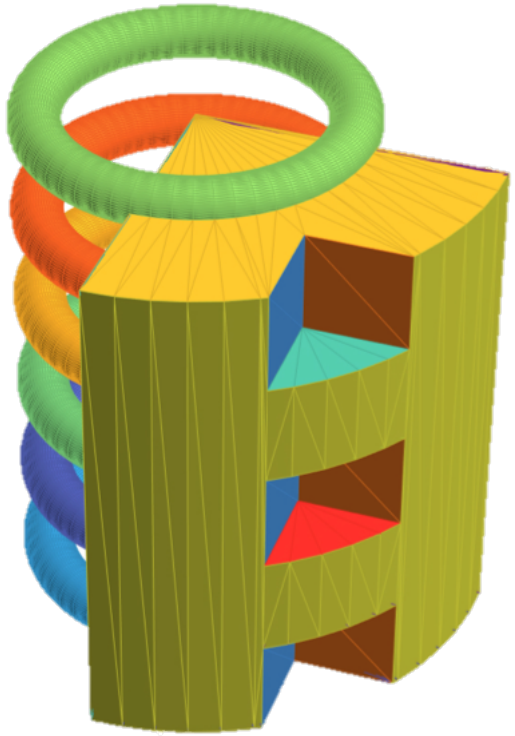
Halfspace fitting



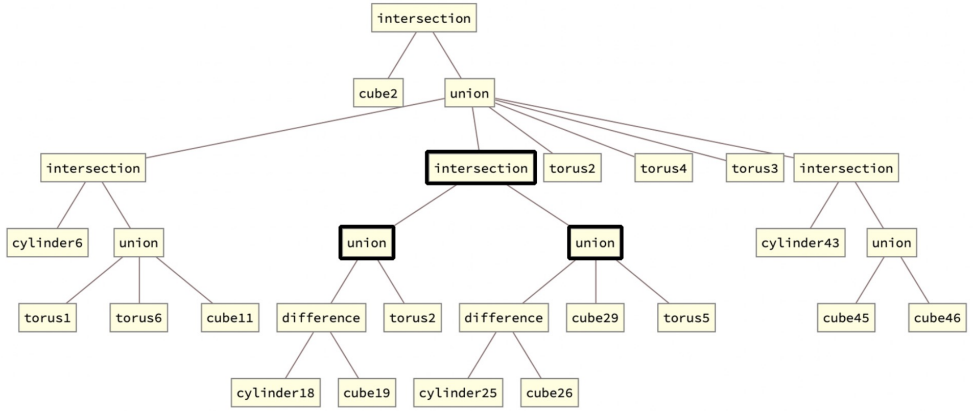
Sample generation

- plane
- sphere
- cylinder
- VIPSS [Huang et al. 2019]

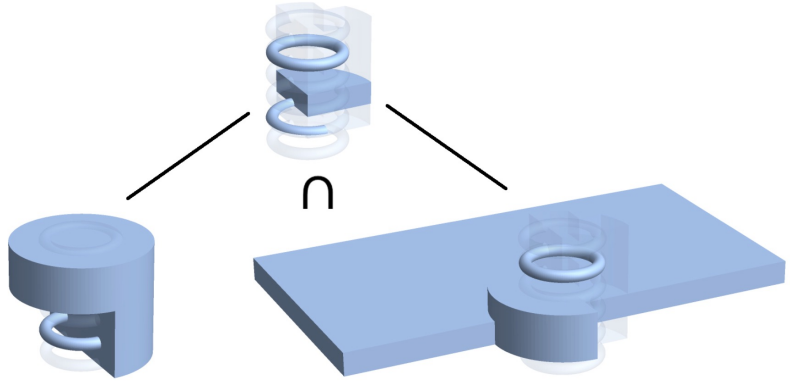
REVERSE ENGINEERING: COMPARISON



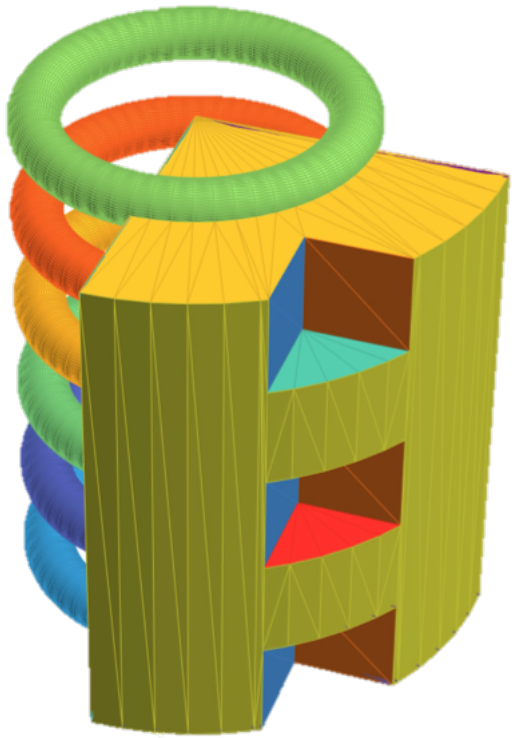
Halfspaces



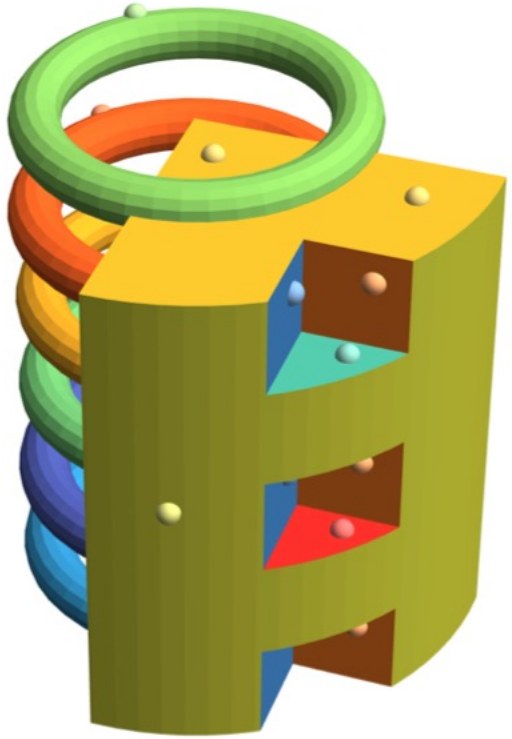
Inverse CSG
[Du et al. 2018]



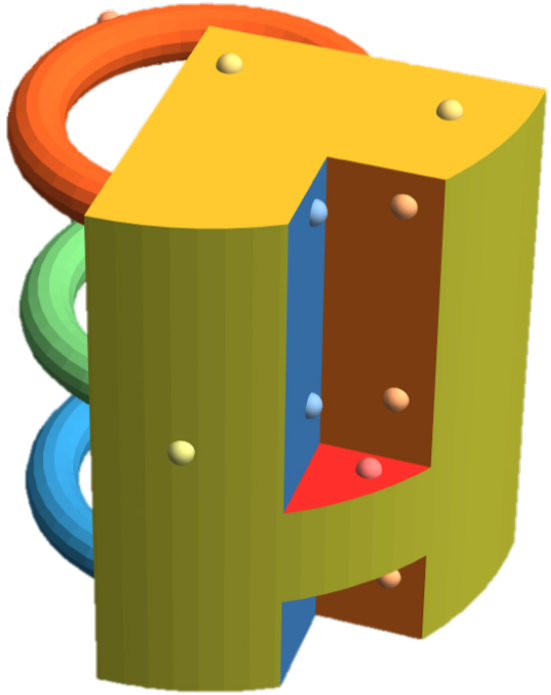
REVERSE ENGINEERING: COMPARISON



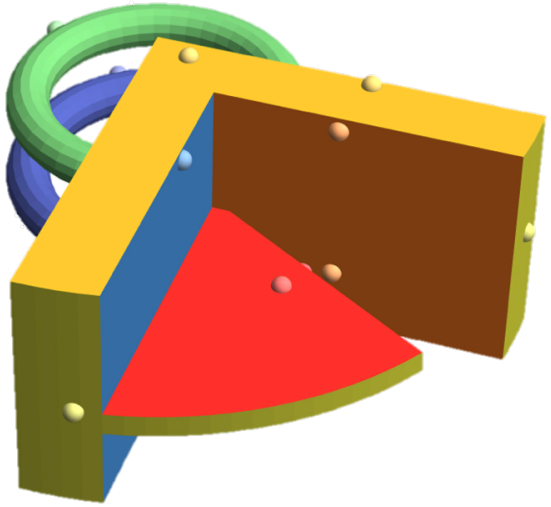
Halfspaces



BSH

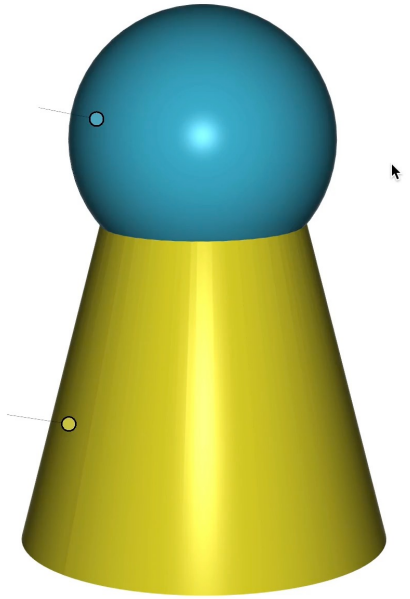


Edit 1

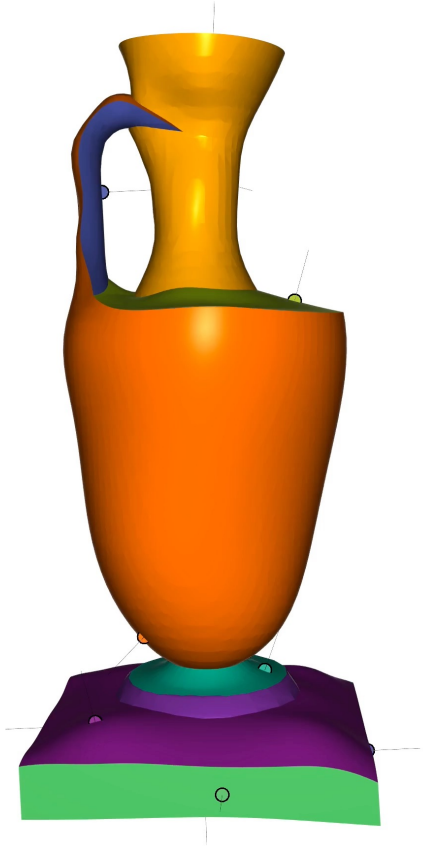


Edit 2

INTERACTIVE EDITING



INTERACTIVE EDITING



SUMMARY

- A new representation for solid models from halfspaces
  - No need for hidden halfspaces
  - Intuitive and light-weight
  - Easy to reverse engineer
- Technical contributions
  - Theoretical analysis of properties
  - Algorithm for boundary extraction and reverse engineering
- Limitation: scalability (due to arrangement computation)

Thanks!

Video, code, program and more at
https://duxingyi-charles.github.io/publication/boundary-sampled-halfspaces/